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Abstract

We study the allocation of German lawyers to different regional courts for their compulsory legal traineeship. The number of applicants exceeds the number of available positions in a given time period in some regions, so that not all lawyers can be matched simultaneously. As a consequence some lawyers have to wait before they obtain a position. First, we analyse the currently used Berlin mechanism and demonstrate that it is unfair and that it does not respect improvements. Second, we introduce a matching with contracts model, using waiting time as the contractual term, for which we suggest an appropriate choice function for the courts that respects the capacity constraints of each court for each period. Despite the failure of the unilateral substitutes condition, under a weak assumption on lawyers' preferences, a lawyer-optimal stable allocation exists. Using existing results, we can show that the resulting mechanism is strategy-proof, fair and respects improvements. Third, we extend our proposed mechanism to allow for a more flexible allocation of positions over time.

Keywords: Many-to-One Matching; Matching with Contracts; Stability, Slot-Specific Priorities, Waiting Time, Legal Education

JEL Classification: D47, D82, C78, H75, I28

1   Introduction

Many real world matching markets fail to match all participants. Those who are unmatched may either leave the market altogether or wait and participate in a later matching procedure. The example that we study here focuses on the allocation of graduating lawyers to their legal traineeship at courts in Germany. In this market congestion arises because of excess demand for positions in some parts of the country. This congestion is managed by requiring unmatched applicants to enter a wait list for their traineeship. To ensure that lawyers will eventually obtain a position at a court, the priority of a lawyer increases with the acquired waiting time.

The education of lawyers in Germany consists essentially of two parts. The first part is the academic training which takes place at universities with a duration of roughly four years. Having completed this, students graduate by passing the First State Exam (*Erstes Staatsexamen*) which is a centralized test for all students happening twice a year. The second part of the education consists of a post-graduate traineeship which typically lasts two years. For this, given their grade in the First State Exam, young lawyers apply to one of the Upper Regional Courts (*Oberlandesgericht, OLG*) to be allocated a position as a trainee (*Referendar*) at a Regional Court (*Landgericht*).

The focus of this work is the traineeship allocation problem between graduated lawyers on the one side and courts on the other side. This is a highly relevant market as each year more than 8,000 lawyers (and around 30,000 teachers) start their post-graduate traineeship in Germany. These numbers are comparable to the (roughly) 20,000 US hospital residency program matches per year studied by Roth and others, e.g. in Roth (1984). Typically there are two to four dates per year at which traineeships begin. In this market the wage is regulated so it cannot be used to reduce congestions by balancing excess demand.

We first study the currently used two-step allocation procedure, which we call Berlin Mechanism although it is used in different variants all over Germany. The details of the procedure are encoded in the law covering the training of lawyers of each *Bundesland*. In the first step, based on priorities derived from different quotas, the mechanism determines the set of lawyers who will be allocated in the given period, without considering any lawyer preferences. Typically a certain percentage of positions are granted by First State Exam grade, some other by waiting time, while some positions are reserved to students meeting social hardship criteria. All lawyers not admitted in a period have to wait for the next period whereby they increase their waiting time which in turn increases their priority subsequently. In the second step, the period's admitted lawyers are allocated to courts based on their reported preferences over courts and their priorities at these courts. While the relevant laws of each *Bundesland* do not contain specific provisions on the matching algorithm used in the second step, we assume the lawyer-proposing Gale-Shapley deferred-acceptance mechanism is used. This is supported by conversations we have had with officials involved in the process.[1] The use of some other procedure in the second step would likely only yield worse performance. Nevertheless, the Berlin Mechanism is not a direct mechanism since lawyers' full preferences over courts and

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[1] We base this on the officials' descriptions of the procedure that they use.

time are not taken into account. As a result the Berlin Mechanism is not fair in the sense that some highly ranked lawyer may envy the assignment of a lower ranked lawyer. Furthermore, a lawyer may receive a worse assignment when her ranking improves. To see this, consider a lawyer who improves her grades. This could lead to her beeing assigned in some earlier period at some rather unpopular court, although her preference would have been to rather wait for a position at a more popular court. Yet, by construction the Berlin Mechanism has one desirable property: it fills court positions early. This means that no lawyer is allocated a position at some court in a future period while leaving open a past position that would have been feasible to her.

To analyse the market while accounting for waiting time as a contract term, we propose a lawyer-court matching problem and set up a matching with contracts model, based on Hatfield and Milgrom (2005). Other related papers are Hatfield and Kojima (2010), Kominers and Sönmez (2013), Sönmez (2013) and Sönmez and Switzer (2013). On the one side of the market there are lawyers, graduating in successive periods, who have preferences over assignments to courts over time. Courts on the other side have priorities over lawyers, based on their grade and time of graduation, which together with the current time period determines a lawyer's waiting time. A matching mechanism in this context produces an allocation consisting of contracts, which specify a lawyer, a court and the time period the traineeship begins.

We propose the time-specific choice function, which is a special case of choice functions based on slot-specific priorities of Kominers and Sönmez (2013). Here time-specific means that each court can only accept a fixed number of students to begin their traineeship in a given period. Using the time-specific choice functions, the cumulative offer process of Hatfield and Milgrom (2005) is used to find stable allocations. When lawyers, for a given court, prefer earlier assignments, then we show the existence of a lawyer-optimal stable allocation. Kominers and Sönmez (2013) do not consider cases in which such a matching exists. In cases when lawyers' preferences are unrestricted, no such lawyer-optimal stable allocation need exist.

The time-specific choice function does not satisfy many properties used in the previous literature. Notably it fails to satisfy the unilateral substitutes and the law of aggregate demand condition. Hence we cannot use the results of Hatfield and Kojima (2009) and Hatfield and Kojima (2010). Under unrestricted preferences we may however apply the results of Kominers and Sönmez (2013) to find that the time-specific lawyer proposing mechanism is (group) strategy-proof for the lawyers. Furthermore this mechanism is fair and respects improvements. Unlike the Berlin Mechanism, the time-specific lawyer proposing mechanism does not ensure that positions are filled early.

We consider another modified version of the matching with contracts model, in which we no longer have time-specific constraints for each court. Instead, courts face only aggregate capacity constraints and are able to shift their positions flexibly over time. This would be applicable, if courts had control over their own budgets over a period of some years. Accordingly we construct the flexible choice function for courts and show that it satisfies the substitutes condition as well as the law of aggregate demand. Hence we can directly apply result from previous work of Hatfield and Kojima (2010) to demonstrate that using these choice functions in a cumulative

offer process yields a lawyer-optimal stable allocation. The resulting flexible lawyer-optimal stable mechanism (FLOSM), like before, is strategy-proof, fair and respects improvements. We find that a welfare ranking of the FLOSM and the time-specific lawyer proposing mechanism is not generally possible as depending on the setting each mechanism might Pareto dominate the other. However, given acyclicality of court priorities over lawyers (Ergin, 2002), we can show that the FLOSM is never Pareto dominated by the time-specific lawyer proposing mechanism.

Last, we suggest how one may further employ slot-specific priorites to allow for more general choice functions for the courts.

While our model has been developed with the entry-level labour market for lawyers in Germany in mind, there are potentially many more applications of the basic framework. For example, university admissions in Germany for some very competitive courses, such as medicine, often ration places by putting unsuccessful applicants on waiting lists. A certain fraction of all seats is then reserved for those applicants who have waited a sufficient number of periods. Another potential application concerns the allocation of aspiring teachers to teaching traineeship positions at schools, in a system very similar to that of lawyers. The main difference to the market for lawyers is that teachers differ based on their chosen subjects, so that schools' preferences over teachers will be more complex than courts' preferences over lawyers, who, by the time of their traineeship, are still essentially homogeneous. Further interesting applications of matching with waiting times are (social or student) house allocation problems or organ transplant problems.

The remainder of this paper is organised as follows. In section 2 we embed our idea into the relevant literature, while in section 3 we give an overview of the German legal education in order to provide a better understanding of the setting. The model is introduced in section 4, where we characterise our matching problem as well as important definitions. Using our model framework, in section 5 we analyse the currently applied Berlin Mechanism and demonstrate its deficiencies. Then, in section 6 we propose and analyse our matching with contracts models. Section 7 concludes, while the Appendix contains proofs that are not in the main text.

2 Literature

This paper fits into the research agenda started by Gale and Shapley (1962) on two-sided matching. For a summary of research in this vein until 1990, see Roth and Sotomayor (1990). More recently a number of papers have applied the original two-sided matching problem to the allocation of seats at universities, for instance Balinski and Sönmez (1999) and, more prominently, to the issue of school choice, Abdulkadiroğlu and Sönmez (2003).

This paper is closely related to the literature on matching with contracts. The canonical model is due to Hatfield and Milgrom (2005), who show that for the existence of a stable matching with contracts, the preferences of hospitals need to satisfy a substitutability condition. Their work is extended by Hatfield and Kojima (2009) who show that under the same conditions as in Hatfield and Milgrom (2005), the doctor-optimal stable mechanism is group strategy-proof. It has been shown recently by Hatfield and Kojima (2010) that the substitutability condition

is not necessary for the existence of stable matchings. Previously the main application of the matching with contracts model has been to labour markets, where a contract specifies a hospital, a student and a wage at which the student is employed by the hospital. This was first studied by Crawford and Knoer (1981) and then extended by Kelso and Crawford (1982). In two recent contributions by Sönmez and Switzer (2013) and Sönmez (2013), another application of the matching with contracts model was found in the way that the US Army allocates positions in different branches with associated minimum service lengths to new cadets. Crucially the choice functions that Sönmez (2013) and Sönmez and Switzer (2013) use do not make use of the older substitutes condition, but instead use the weaker unilateral substitutes condition of Hatfield and Kojima (2010). This paper provides a practical application of the matching with contract framework, where the waiting time until a position can be taken up is now a term of the contract between a student and a court. Unlike Sönmez (2013) and Sönmez and Switzer (2013) the choice functions that we construct in this paper do not satisfy the unilateral substitutes condition, but only the weaker bilateral substitutes condition. Our work is also closely related to Kominers and Sönmez (2013) who study a more general slot-specific matching with contracts model. In fact we make use of their results in order to show strategy-proofness and respect of improvements in the absence of unilateral substitutes and the law of aggregate demand condition being satisfied. However, unlike their study, we can show the existence of a lawyer-optimal stable mechanism by using our weak impatience assumption. Aygün and Bo (2013) study college admissions with affirmative action in Brazil. In Brazil members of minorities and poor students benefit from affirmative action on their behalf. To be eligible for affirmative action students need to report verifiable information concerning their status. Hence a mechanism needs to give students the right incentives to report such information. The authors show that the currently used mechanism does not do so and suggest a mechanism which does. Incentivising students to reveal verifiable information is also a concern in our paper.

In a recent paper, Hirata and Kasuya (2014) show that under the bilateral substitutes condition, the order in which contracts are offered in the cumulative offer process does not affect its outcome. Another paper by Flanagan (2014) weakens the bilateral substitutes condition. He considers choice functions that during the cumulative offer process behave as if they satisfied the bilateral substitutes condition. In our extension to a more flexible lawyer-optimal stable mechanism we borrow the acyclicality result from Ergin (2002) who finds conditions for the existence of a Pareto efficient outcome.

Another related literature is the one on dynamic matching markets. Papers in that literature have, to our knowledge, not yet made use of the matching with contracts framework. Damiano and Lam (2005) consider one-to-one matching markets which are repeated over time. Here the outcome is a matching associating one man to a woman for each period. Similarly, Kurino (2009) considers one-to-one repeated matching markets. The focus in the latter paper is on a new notion of credible group-stable dynamic matchings. The paper by Bloch and Houy (2012) considers the allocation of a set of durable objects to agents who successively arrive and live for two periods. He characterises a stable allocation procedure and shows some properties of this. Related, Kurino (2014) considers a dynamic house allocation problem in which agents arrive successively and live for two periods. Abdulkadiroğlu and Loertscher (2007) also consider a dynamic house allocation problem. That paper compares static and dynamic mechanisms,

finding that the latter can improve welfare upon the former. Another market design application of dynamic matching problems is Kennes et al. (Forthcoming) who consider the allocation of small children to daycare facilities in Denmark. Our paper differs from these paper insofar as in our paper the outcome is a set of contracts in which each lawyer appears only once, so no lawyer is matched repeatedly. Also, unlike the previous papers we make explicit use of the matching with contracts literature, which might also be fruitfully applied in the papers just mentioned. To apply the matching with contracts framework one would simply need to allow the lawyer side to hold multiple contracts as long as they refer to different time periods.

This paper is also related to some papers within the theory of matching which analyse different legal entry-level labour markets. In Avery et al. (2007), the authors describe the unravelling in the market for legal clerkships at US federal courts for graduating law students. There the problem is that exploding offers are made. Notably this market is a decentralised one with no central authority designing an allocation procedure. Additionally, there is some conflict among the judges which prevents an effective coordination to improve the system. In contrast, the market for legal traineeships in Germany is highly centralised and in fact mandated by law. Issues of market unraveling are absent in the German case studied in this paper. This study, in comparison with the work of Avery et al. (2007) could thus improve our understanding of how to limit unravelling, as analysed in for example Roth and Xing (1994), Niederle and Roth (2003) and Niederle and Roth (2009). In this vein, this paper also related to the market design literature, e.g. Roth and Peranson (1999), which tries to understand and help improve currently used mechanisms.

Two further related papers are Schummer and Vohra (2013) and Schummer and Abizada (2013). The former paper considers the assignment of landing slots to planes in the event of adverse weather. It shows the lack of incentives to report truthfully the estimated arrival times for flights under the currently used mechanism and proposes a strategy-proof alternative. However neither of the two mechanisms studied gives airlines incentives to report cancelled flights. The latter considers in more detail incentive problems in landing slot problems. That paper also highlights the restrictions that notions of incentive compatibility impose on the efficiency of the resulting mechanisms. The landing slot allocation problem as studied in those papers also differs from the lawyer allocation problem studied here. First, the paper assumes that all future arrival times are known by the airlines at the time an allocation is made. Second, the airlines have homogeneous preferences for early arrival at a single airport. So unlike in the present paper, there is only one good to be allocated in any time period.

Last, this paper is also related to a small literature concerning matching problems within Germany. The only two examples that we are aware of are Braun et al. (2010), who study the mechanism used to allocate medical students to universities. Students in that mechanism are twice asked to submit preferences without any constraint of consistency among the two reports. The authors show strong incentives to misrepresent preferences in the first part of the mechanism. Similar to the allocation of lawyers to courts, the issue of waiting time is also relevant for the allocation of medical students to universities, since a fraction of slots is reserved for students according to the time they have waited for a position. This stems from a ruling by Germany's Constitutional Court, declaring that access to medical school should not be denied for holders of the German high-school diploma. However the authors in that paper do not focus

on the waiting time issue. Second, Westkamp (2013) further analyses the allocation of medical students to universities. He introduces matching with complex constraints and finds equilibria of the game induced by the currently used mechanism. Like Braun et al. (2010), Westkamp (2013) does not address the waiting time issue in the allocation of university places.

3 Overview of German Legal Education

Unlike in the United States, in Germany lawyers typically begin their legal education as an undergraduate, studying law at a university for around four years. Afterwards students take a First State Exam (*Erstes Staatsexamen*), set by each of the 16 *Bundesländer*. Following this, students apply for a period of practical legal education, a legal traineeship (*Juristischer Vorbereitungsdienst* or *Referendariat*). Having completed their traineeship, which usually takes around two years, young lawyers need to pass a Second State Exam (*Zweites Staatsexamen*) after which they are eligible to practice law in Germany and become a *"Volljurist"*. While the first part of a lawyer's legal education takes place at a university, the second part is mainly organised by one of the Upper Regional Courts (*Oberlandesgerichte, OLG*). During a typical legal traineeship a lawyer has to spend several months at different stations, including a Court for Civil matters, a Court for criminal matters, a lawyer's office as well as some agency of the executive branch of government. Usually, during the traineeship the lawyer can also visit a position of their choice once the compulsory stations have been completed. Parallel to working for these different courts and offices, lawyers spend a significant part of their time furthering their legal education in special seminars in preparation for the Second State Exam.

While the system of clerkships at federal courts in the US developed informally, the German legal traineeship is a compulsory part of a lawyers' education. A lawyer can only become a judge at a German court if some threshold grade has been reached in the Second State Exam (this is called a *Prädikatsexamen*). Each of the *Bundesländer* has a specific law regulating the content and form the traineeship takes as well as setting both State Exams.

The number of available positions for the traineeship varies by court and usually depends on its size and the budget that has been made available for legal trainees in the budget of the *Bundesland*. The budget determines how many positions are available at a court in each intake period. Currently more than 8000 legal trainees are assigned each year (see Table 1). In a period, the number of available positions over all courts does not depend on the number of positions sought by lawyers applying for the legal traineeship. As a result, in several *Länder* (notably Northrhine-Westphalia, Hamburg and Berlin) not all lawyers applying for a position at a court can be allocated a position. In that case, in most *Länder* the excess demand is managed via a system involving waiting times. Whenever a lawyer could not be successfully matched to a position, he will usually be taken to automatically apply to a position at the next intake time. Most *Länder* have a system whereby a lawyer's priority in being allocated a place at a court increases in the number of intake times that lawyer was not matched. Thereby it is in principle possible for each lawyer to gain some place in a *Bundesland* eventually. This is because a greater waiting time will improve the ranking of a lawyer. At the moment, waiting times are usually less than a year, while in some (larger) States there are none at all (see Table 1).

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Table 1: Overview of allocation procedure by German Federal Land

Federal Land	Step 1 priorities	Positions in 2013	Waiting time avg. in 2013	Upper R. Courts
Baden-Württemberg	1) 65% by $[G + 0.5 \times W]$ 2) rest by W 3) 10% by H	550	0	3
Bayern	unclear	1440	0	3
Berlin	1) 20% by G if $G \geq 10$ 2) 10% by H 3) rest by* W	720	7-13 months	1
Brandenburg	1) 20% by G 2) 10% by H 3) rest by W	160	2 months	1
Bremen	1) 15% by H 2) 45% by** $[G + W]$ 3) rest by G	60	n/a	1
Hamburg	1) all by $min\{G, 6.49\} + W$	600	24 months	1
Hessen	unclear, by G and H	780	0	1
Mecklenburg-Vorp.	1) 35% by G 2) rest: 10% by H, 90% by W	80	0-6 months	1
Niedersachsen	1) by H; 2) by $[G + W]$	520	0-3 months	3
Nordrhein-Westfalen	1) by H 2) by W	2000	2-4 months	3
Rheinland-Pfalz	1) 20% by H 2) 60% by G 3) rest by W	380	0	2
Saarland	1) all with $W \geq 2$ years 2) 10% by H 3) rest: 60% by G, 40% by W	70	6 months	1
Sachsen	unclear, by H	180	0	1
Sachsen-Anhalt	1) all with $W \geq 2$ years 2) rest: 45% by G, 40% by W 3) by H	60	0	1
Schleswig-Holstein	1) 20% by G 2) 10% by H 3) rest by W	360	0	1
Thüringen	1) all with $W \geq 3$ years 2) 40% by G 3) 10% by H 4) rest by W	100	n/a	1
		$\Sigma = 8020$		

"G" = First State Exam *grade* (between 1 and 18), "W" = *waiting time* in unsuccessful application periods, "H" = *hardship* criteria

Under Step 1 the order of applied priority criteria as well as their respective (maximal) quota in percentage of total capacity is listed. Note that if a certain quota, for instance often the hardship quota, is not reached, then capacity is filled based on the other criteria.

* Berlin step 3): 80% to students with First State Exam from a Berlin university, 20% to others

** Bremen step 2): formula in detail: $[G + min\{4, W\} + 2\,max\{0, W - 4\}]$

Source: German Federal State codes on the education of lawyers and www.juristenkoffer.de

Lawyers typically apply to a position at an Upper Regional Court, of which there is usually one per Land. Only larger *Länder* like Bavaria or Northrhine-Westphalia have up to three Upper Regional Courts. The actual positions however are at a lower jurisdictional level, namely at that of the Regional Courts (*Landgerichte, LG*). There are currently no restrictions on applying to Upper Regional Courts in several *Länder* at the same time, although some *Länder* explicitly ask their applicants to state whether they have done so. While applications are made to the a particular Upper Regional Court, those larger *Bundesländer*, such as Bavaria have a system whereby imbalances among the Upper Regional Courts can be balanced within the *Bundesland*, thereby allowing them to reduce waiting times.

When applying for a position lawyers can typically indicate a preference for a particular regional court. For example, lawyers applying to do their traineeship in Brandenburg apply to the only Upper Regional Court of Brandenburg and can be allocated either to the Regional Court in Potsdam, Frankfurt (Oder) or Cottbus. While lawyers may often state preferences for the court they want to be matched to, there is no legal guarantee of being assigned the most preferred court as some courts are typically over-demanded (e.g. Potsdam in the case of Brandenburg or Munich in Bavaria). In some *Länder* lawyers can put forth substantiated reasons for why they should be allocated to a particular court. For example, having dependant children living in a town is often sufficient to ensure a place at the court of that town. This means that courts might have different priorities over lawyers.

The German *Länder* use a two-step procedure for allocating capacities. In the first step, the set of lawyers who will be allocated a place at an intake date is determined. In the second step, lawyers receiving a place at the current intake date are allocated to a regional court. The details of the mechanism for selecting lawyers in each of the two steps varies by Land. In general one can say that in the first step lawyers are more likely to be allocated a place if they have spend more time waiting to be allocated a seat, if their performance in the First State Exam is better and, in some *Länder*, if they completed their first few years of legal education at a university in the same Land. In the second step of the allocation procedure, those lawyers to be allocated a place are distributed among the regional courts. The second column in Table 1 provides an overview of the first step allocation priorities in the different *Länder*. While the law sometimes provides that lawyer preferences should play a role, the procedure is not made explicit in any of the *Länder*[2]. Once lawyers are allocated to a Regional Court they are often given the option to reject that allocation. Such a rejection usually means the lawyer drops out of the process and has to apply again (losing all accumulated waiting time, if any). If some lawyers reject their allocated place, then other lawyers in order of their priority from the first step of the procedure, are used to fill up the remaining places.

4 Model

This section introduces the lawyer assignment problem. We abstract from problems arising from the fact that lawyers arrive sequentially over time and focus on the case in which a given set of lawyers is to be allocated to courts over several time periods. Each court can only accept a

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[2]Informal conversations with people familiar with the process suggest that there is no clear procedure for allocating lawyers to the *Landgerichte*. Allocations seem to be made on an ad-hoc basis using "common-sense".

fixed number of lawyers per period. The presentation follows Sönmez (2013).

The lawyer assignment problem consists of the following components:

1. a finite set of periods $T = \{1, ..., T\}$

2. a finite set of lawyers $I = \{i_1, ..., i_n\}$

3. a finite set of courts $C = \{c_1, ..., c_m\}$

4. a matrix of court capacities $q = (q_{c,t})_{c \in C, t \in T}$

5. lawyers' (strict, rational) preferences $P = (P_i)_{i \in I}$ over $C \times T \cup \emptyset$, with R_i denoting weak preferences of lawyer i. We place no restriction on the domain of preference profiles, denoted $\mathcal{P}$.

6. a list of courts' priority rankings, $\succ = (\succ_c)_{c \in C}$ over I. These can be thought of as a single score as a function of a lawyer's grade, waiting time and social factors, such as place of birth, current residence or place of study. Since we consider a static setting, we will not consider how these priority rankings might change.

A **contract** is a triplet $x = (i, c, t) \in I \times C \times T$, specifying a lawyer, a court and the time at which the lawyer begins her traineeship at the court. Let $X \subseteq I \times C \times T$ be the set of all feasible contracts. A contract x is **acceptable** to lawyer i if $xP_i\emptyset$.

For contract $x = (i, c, t)$ we denote by x_I the lawyer appearing in x, i.e. $x_I = i$. Similarly we denote by x_C and x_T the court and the time period of assignment appearing in contract x, i.e. $x_C = c$ and $x_T = t$. Further, let Y_I be the set of lawyers appearing in some set of contracts $Y \subseteq X$, that is $Y_I = \{i \in I \mid \exists y \in Y \, s.t. \, y_I = i\}$.

A set of contracts $Y \subseteq X$ is an **allocation** if for all $i \in I$, $|\{y \in Y : y_I = i\}| \in \{0, 1\}$ and for all $c \in C$ and $t \in T$, $|\{y \in Y : y_C = c \cap y_T = t\}| \leq q_{c,t}$. In words, an allocation is a set of contracts such that no lawyer appears more than once and there are not more contracts of a court for some period than number of positions available at that court in that period. Let $\tilde{X}$ be the set of allocations. Denote by $Y(i)$ for $Y \in \tilde{X}$ the unique contract involving lawyer i that is part of allocation Y, or alternatively, if i has no contract in Y then $Y(i)$ is the empty set. Furthermore, let $Y_T(i)$ be the time of start of traineeship according to i's contract in Y. We define $Y_C(i)$ accordingly.

We denote by P_i not only preferences over i's assignment of a court and a time period, but also i's preferences over allocations. These preferences over allocations reflect i's preferences over assignments, so there should be no loss of clarity in this abuse of notation.

A **direct mechanism** ψ is a function $\psi : \mathcal{P} \to \tilde{X}$. Hence ψ associates to each (reported) preference profile an allocation. Note that we treat courts as objects and hence they do not behave strategically, i.e. their priorities over lawyers are assumed to be given. In full generality the mechanism should also depend on the number of available places in each court for each period as well as the rankings the courts have over the lawyers. We suppress this dependence for simplicity but will highlight whenever it becomes relevant, for example when comparing the outcome of some mechanism when a court's ranking of the lawyers has changed.

We now describe a few properties that lawyer preferences over the courts and the time of allocation can satisfy.

Definition. [Time Independence] Preferences of lawyer $i \in I$ satisfy **time independence**, if for all $c, \tilde{c} \in C$ and for any $t, t' \in T$, then $(c, t) R_i(\tilde{c}, t)$ implies $(c, t') R_i(\tilde{c}, t')$.

A lawyer's preferences satisfy time independence if a lawyer does not change his ranking over the courts over time. So if a lawyer prefers court c to $\tilde{c}$ in period t, then she should also prefer c to $\tilde{c}$ in period t'.

Definition. [Weak Impatience] Preferences of lawyer $i \in I$ satisfy **weak impatience** if for all $c \in C$, $t, t' \in T$ such that $t < t'$, then $(c, t) R_i(c, t')$.

A lawyer's preferences satisfy weak impatience if a lawyer prefers to be allocated an early position at some court to a later position at the same court.

Definition. [Strict Impatience] Preferences of lawyer $i \in I$ satisfy **strict impatience** if for all $c, \tilde{c} \in C$ $t, t' \in T$ such that $t < t'$, then $(c, t) R_i(\tilde{c}, t')$.

Strict impatience is a strengthening of weak impatience. A lawyer having strictly impatient preferences prefers an early position at any court to a later position at *any* court. Note that when lawyers' preferences satisfy both time independence and strict impatience then it is possible to construct the preferences of the lawyers over the whole domain from their preferences over just the courts.

4.1 Properties of Allocations and Mechanisms

To analyse the outcome of different mechanisms it is necessary to be able to talk about properties of allocations. We begin by discussing the notion that whenever a position is not filled in some period, then no agent who would have been available that period should be assigned later. It seems reasonable to suppose that policy-makers would not be willing to allow some place at a court to go unfilled simply to allow a current applicant to obtain an allocation to a preferred court in a later period. This is first because lawyers provide essential work to the court at the time of their traineeship and second because like this more future slots are left open which makes future lawyers (weakly) better off.

Definition. [Early Filling] An allocation $Y \subseteq X$ satisfies **early filling** if there is no $t \in T$ such that there exists some $c \in C$ such that $|\{y \in Y : y_T = t, y_C = c\}| < q_{c,t}$ and there exists some $i \in I$ such that $Y_T(i) > t$. A mechanism ψ satisfies early filling if for all $P \in \mathcal{P}$, $\psi(P)$ satisfies early filling.

We next introduce a common notion of fairness:

Definition. [Fairness] An allocation $Y \subseteq X$ is **fair**, if for any pair of contracts $x, y \in Y$ with $x_I \neq y_I$ and $(x_C, x_T) P_{y_I}(y_C, y_T)$, then $x_I \succ_{x_C} y_I$. A mechanism ψ is fair if its outcome $\psi(P)$ is fair for all $P \in \mathcal{P}$.

An allocation thus is fair if, whenever a lawyer prefers some other lawyers' assignment, then that lawyer must have a higher priority at the court she is being assigned to than the former lawyer.

The following definition of Pareto efficiency is standard.

Definition. [Pareto dominated] An allocation $Y \subseteq X$ is **Pareto dominated** by another allocation $\tilde{Y} \subseteq X$ if for all $i \in I$ $\tilde{Y}(i) R_i Y(i)$ and there exists at least one $i \in I$ such that $\tilde{Y}(i) P_i Y(i)$. A mechanism ψ Pareto dominates another mechanism $\tilde{\psi}$ if for all $P \in \mathcal{P}$ $\psi(P)$ Pareto dominates $\tilde{\psi}(P)$. An allocation is **efficient** if it is not Pareto dominated by another allocation.

As usual, a mechanism is strategy-proof if it is a dominant strategy for each agent to truthfully report her preferences to the mechanism:

Definition. [Strategy-Proofness] Mechanism ψ is **strategy-proof** if for all $i \in I$, for all $P \in \mathcal{P}$ and for all $\tilde{P}_i \in \mathcal{P}_i$ we have $\psi(P) R_i \psi(\tilde{P}_i, P_{-i})$. Mechanism ψ is **group strategy-proof** if, for any preference profile $P \in \mathcal{P}$, there is no $\tilde{I} \subseteq I$ and $\tilde{P}_{\tilde{I}} = (\tilde{P}_i)_{i \in \tilde{I}}$ such that for all $i \in \tilde{I}$ we have $\psi(\tilde{P}_{\tilde{I}}, P_{-\tilde{I}}) P_i \psi(P)$.

We next state another property, first used in the matching literature by Balinski and Sönmez (1999), that of respect of improvements. What that property means is that a lawyer should not receive a worse assignment when her priority has increased at the courts. First we need to define what we mean by an improvement in the priority of a lawyer. In doing so, we will follow closely the presentation in Sönmez (2013).

Definition. [Priority profile improvement] A priority profile $\succ$ is an **unambiguous improvement** over another priority profile $\succ'$ for lawyer i if:
    - the ranking of lawyer i is at least as good under $\succ$ as $\succ'$ for any court c,
    - the ranking of lawyer i is strictly better under $\succ$ than under $\succ'$ for some court c,
    - the relative ranking of other lawyers remains the same between $\succ$ and $\succ'$ for any court c

Intuitively, a priority profile improvement of some lawyer means that while all other lawyers' relative rankings among the courts are unchanged, the particular lawyer's ranking is not worse at any court (i.e. there are at most as many lawyers ranked higher than the lawyer as before) and the lawyer's ranking has improved at least at one court.

Definition. [Respect of improvements] A mechanism ψ **respects improvements** if a lawyer never receives a strictly worse assignment as a result of an unambiguous improvement in her court priorities.

Respect of improvements is a natural property to ask for. Suppose that a better grade for a lawyer leads to an unambiguous improvement in that lawyer's ranking by the courts. Then, if respect of improvements did not hold, the lawyer would have received a less preferred position than with the worse grade. This would run counter to the view that law students should be rewarded for good performance in the exams. In addition, some may consider it to be unjust that lawyers obtain a better outcome for themselves despite having a worse grade, compared to another lawyer.

More important, perhaps, is the implicit reliance of existing procedures on waiting time in ranking lawyers. Suppose that under some specified mechanism a lawyer improves her ranking

by arriving earlier, then, if the mechanism tries to aid lawyers who arrive early by improving their ranking, this attempt to increase the welfare will hurt those lawyers if the overall mechanism does not respect improvements.

5 Berlin Mechanism

We now study the procedure that is currently used in Germany to allocate lawyers to courts. While some aspects of that procedure are well documented, the part describing how lawyers are allocated to courts within a period is not. Based on conversations with officials involved in the process we assume that the Deferred-Acceptance algorithm of Gale and Shapley (1962) is used.

The following set-up is stylized for one artificial *Land* since we want to place emphasis on how the employed allocation procedure works and then show its deficiencies. Of course, a major inefficiency is the absence of a centralized matching system over the whole of Germany but this will not be our focus.[3]

In order to describe the Berlin Mechanism, we will make use of the total capacity over all courts in time t, which is $Q^t = \sum_{c=1}^{|C|} q_{c,t}$. In the first step, where the set of admitted lawyers is found, the lawyer priorities of the court-side can be simplified due to priority quotas, which are imposed by the law for the whole set of accepted lawyers over all courts in each year. Let these quotas λ_G and λ_W be the fraction of seats to be given to lawyers based on state exam grades and on waiting time respectively. We assume $\lambda_G + \lambda_w = 1$, i.e. we abstract from hardship or other quotas here.

The Berlin Mechanism then proceeds as follows:

- **Step 1a:** Select up to $\lambda_G Q^t$ lawyers with the highest scores in the First State Exam, while breaking ties using waiting time, age and lottery.

- **Step 1b:** From the set of lawyers not yet selected, select up to $\lambda_W Q^t$ lawyers with the highest waiting time, while breaking ties using the grade in the First State Exam, age and lottery.

- **Step 2a:** All lawyers not yet selected are placed on hold and provisionally have to wait until the next period, whereby their waiting time increases by one.

- **Step 2b:** Apply the GS-DA algorithm on the (previously submitted) preferences of the lawyers who have so far been selected and on the courts' priorities. Assign each lawyer to the court assigned under this algorithm.

In addition to the previous description of the allocation procedure, there are additional peculiarities that may affect its performance, which for now we abstract from in the following theoretical discussion. First, truncated court preferences: In some *Länder*, only two more courts in addition to the most-preferred one can be reported (sometimes with no ordering possible) and if the lawyer is not allocated to any of these three, then her preference list is randomly filled with non-listed courts. Second, endogenous court priorities: lawyers can report a *verifiable* special social connection to some courts, e.g. a spouse or other relatives living in that region

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[3]It should be noted that in *Länder* with more than one Upper Regional Court - these are six out of sixteen (see Table 1) - each Upper Regional Court district could be interpreted as a single *Land* in our setting.

etc., leading to higher priority at that court. Third, refusals to accept positions: lawyers are informed of their allocated court, but they can refuse to accept that position. Refusing lawyers are replaced by those still on the waiting list. Usually, refusals lead to non-accrual of waiting time.

5.1 Deficiencies of the Berlin Mechanism

The algorithm as currently used has a number of flaws, mainly associated to the fact that it is not a direct mechanism and that preferences are not considered when determining which lawyers are to be allocated in a given time period. This is because lawyers can only announce a preference order over the courts. This makes it impossible to express preferences over courts and time of allocation. As a result, when lawyers' preferences are unrestricted, it is not even clear what preferences over courts lawyers will report to the mechanism. However, when lawyers' preferences satisfy time independence, this implies that for each lawyer, there is a preference order, which we denote by P_i^C, over courts which is consistent with the lawyers underlying preferences over courts and time. We can now state the following lemma.

Lemma 1. *Suppose lawyers' preferences satisfy time independence. Then for each lawyer, reporting P_i^C is an optimal istrategy.*

Proof. In steps 1a and step 1b of the Berlin Mechanism, reported preferences are unimportant. Then consider any lawyer i who is to be matched in the current period. For this lawyer the only preferences that are relevant are those over courts in that time period, which can be represented by P_i^C since it is consistent with that lawyer's true preferences, because of time independence. Since in Step 2 of the Berlin mechanism, the lawyer-proposing DA algorithm is used, truthfully reporting those preferences is a dominant strategy. The same argument applies to any lawyer who is to be allocated in a later period. □

In the remainder of the section we will assume that lawyers' preferences satisfy time independence. We can then apply Lemma 1 to obtain lawyers' reports in the Berlin Mechanism. Note that restricting attention to preferences that are time independent is without loss of generality. We have the following result.

Proposition 1. *The outcome of the Berlin Mechanism is unfair and does not respect improvements.*

Proof. Consider the following lawyer assignment problem as a proof by example.

Example 1.
[Current algorithm is unfair and inefficient] There are two periods, $t = 1, 2$. We have lawyers $I = \{i, j, k\}$. There are two courts, i.e. $C = \{a, b\}$. $q_{a,1} = q_{a,2} = q_{b,1} = 1$ and $q_{b,2} = 0$. Court priorities are

$$i \succ_c j \succ_c k \quad \text{for all } c \in C.$$

Lawyer preferences are:

$$i: \quad (a,1)P_i(a,2)P_i(b,1)$$
$$j: \quad (a,1)P_j(a,2)P_j(b,1)$$
$$k: \quad (a,1)P_k(b,1)P_k(a,2)$$

In period 1, in the first step the two lawyers with highest priority (i and j), regardless of their preferences, are selected to be allocated to the two open spots in the first period. Lawyer k is put on hold, increases her waiting time, and will be reconsidered in the next period. In the second step of period 1, now based on lawyers' preferences, i and j are matched to their favourite courts, respecting their priority, and using GS-DA mechanism. In period 2, there is only k who is then allocated.

Therefore the Berlin Mechanism produces the following (unique) outcome $X^{Berlin} = \{(i,a,1),(j,b,1),(k,a,2)\}$. This outcome is not fair since there exists justified envy of j, i.e. $(a,2)P_j(b,1)$, although $j \succ_c k$.[4]

[Current algorithm does not respect improvements] Consider the previous set-up. If courts' priority orders are changed to $i \succ_c k \succ_c j$, then the resulting allocation under the Berlin Mechanism is $X^* = \{(i,a,1),(j,a,2),(k,b,1)\}$. Now, if j improves, e.g. with a better grade, such that the old priority ranking as above is recovered, then X^{Berlin} would result and j would be worse off. Hence the algorithm does not respect improvements.

$\square$

We have seen that the Berlin Mechanism is unfair and does not respect improvements for general preferences. One question that could be considered is whether there exist a class of preferences for which the Berlin Mechanism is fair and respects improvements. As it turns out for preferences which satisfy *strict impatience* and time independence the currently used allocation procedure always delivers a fair allocation and respects improvements. This is summarized in the following proposition.

Proposition 2. *Suppose the preferences of each lawyer satisfy strict impatience. Then the Berlin Mechanism is fair and respects improvements.*

While this result is somewhat encouraging, one should note that in practice it is not obvious that lawyers have preferences that satisfy strict impatience. In essence strict impatience implies that lawyers are unwilling to wait a few months to obtain a preferred court over one that they dislike. This does not seem very realistic, although it may hold for some lawyers.

Consider the question of whether the Berlin Mechanism is strategy-proof under the assumption that lawyers' preferences satisfy strict impatience. Then, lawyers are able to truthfully state their preferences since the combined assumptions of strict impatience and time independence imply that the full set of preferences of the lawyers over courts and allocation times can be recovered from a rank-order list over just the courts. Since the allocation to each court within a

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[4]Note that the allocation $X^* = \{(i,a,1),(j,a,2),(k,b,1)\}$ is preferred by j and k and weakly preferred by i to X^{Berlin} and hence Pareto dominates it, despite the fact that courts' rankings of the lawyers are the same.

time period (step two) is the only part of the allocation procedure which is affected by lawyers' stated preferences, it follows immediately that the Berlin Mechanism will be strategy-proof for strictly impatient preferences whenever the algorithm used for the second step is strategy-proof. Since the Gale-Shapley mechanism is strategy-proof, it follows that the Berlin Mechanism is also strategy-proof. We summarize this discussion, which already exhibits the proof, below.

Proposition 3. *Suppose preferences of each lawyer satisfy strict impatience. Then the Berlin Mechanism is strategy-proof.*

One important benefit of the Berlin Mechanism in practice is that it fills court positions early. This can easily be seen by the fact that a lawyer arriving in an early period being assigned in a later period while another lawyer arriving in that later period being unassigned, can only happen if the early-arriving lawyer was not assigned earlier. But given the two-step procedure, if there had been places left earlier, the early-arriving lawyer would have been assigned then. Hence the Berlin Mechanism satisfies *early filling*. Note that this finding holds for all possible lawyer preference profiles, since the assignment to a period, and thus the decision to fill all currently open positions, is determined in the first step of the procedure, which is independent of lawyers' preferences. We summarize this finding in the following proposition.

Proposition 4. *The Berlin Mechanism fills positions early.*

6 Stable Mechanisms

6.1 Choice Functions and their Properties

In the previous section we have seen that the currently employed procedure of allocating lawyers to their traineeships has some serious deficiencies. In this section we propose a procedure which overcomes these problems. Our approach is to first take the court priorities as used in the current procedure and then to construct choice functions, as in the matching with contracts literature. Having constructed the choice functions we can then use the cumulative offer process of Hatfield and Kojima (2010) to find a stable allocation. Specifying appropriate choice functions for the lawyers does not present a difficulty since a lawyer will simply pick her most preferred contract from the set of available contracts. The choice functions for the courts are somewhat harder to define.

We will denote general choice functions of some agent $j \in I \cup C$ as $\mathbb{C}_j$. When we write $\mathbb{C}_i$ then the choice function of an agent $i \in I$ is meant, whereas $\mathbb{C}_c$ denotes the choice function of a court $c \in C$. A lawyer i's choice function $\mathbb{C}_i(Y)$ specifies for each set of contracts $Y \subseteq X$ which contract the lawyer chooses.

$$\mathbb{C}_i(Y) \equiv \max_{P^i} Y$$

The above formulation says that lawyer i will choose from set Y the contract naming lawyer i that is maximal according to the lawyer's preferences P_i. If Y does not contain a contract with i then $\mathbb{C}_i(Y) = \emptyset$. We will make use of the following definitions of unilateral and bilateral substitutes from Hatfield and Kojima (2010):

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Definition. [Unilateral Substitutes] Contracts are **unilateral substitutes** for court c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z_I \notin Y_I$, $z \notin \mathbb{C}_c (Y \cup \{z\})$ and $z \in \mathbb{C}_c (Y \cup \{x, z\})$.

Intuitively contracts are unilateral substitutes for a court if, when some contract of a lawyer who has only one contract in the available set is not chosen, that contract is also not chosen when some other contract is added to the available set.

Definition. [Bilateral Substitutes] Contracts are **bilateral substitutes** for court c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z_I, x_I \notin Y_I$, $z \notin \mathbb{C}_c (Y \cup \{z\})$ and $z \in \mathbb{C}_c (Y \cup \{x, z\})$.

Bilateral substitutes is a less strict requirement on choice functions, since it only requires a rejected contract of an agent who has no other contract in the available set to only also be rejected when another contract of an agent, who so far did not have a contract in the set of available contracts, is added to the set of available contracts.

The following irrelevance of rejected contracts property as defined by Aygün and Sönmez (2012) will be needed:

Definition. [Irrelevance of Rejected Contracts] Choice functions satisfy **irrelevance of rejected contracts** for court c if for all $Y \subset X$ and for all $z \in X \setminus Y$, we have $z \notin \mathbb{C}_c (Y \cup \{z\})$ implies $\mathbb{C}_c (Y) = \mathbb{C}_c (Y \cup \{z\})$.

Irrelevance of rejected contracts simply means that the availability of contracts which are not chosen does not matter for choices.

Although we will rely on the results of Kominers and Sönmez (2013) to establish strategy-proofness of the cumulative offer process, another way is to use the law of aggregate demand, first introduced by Hatfield and Milgrom (2005):

Definition. [Law of Aggregate Demand] The choice function of court $c \in C$ satisfies the **law of aggregate demand** if for all $X' \subseteq X'' \subseteq X$, $|\mathbb{C}_c (X')| \leq |\mathbb{C}_c (X'')|$.

For court c the model set-up does not necessarily imply a particular choice function. Therefore we will specify a particular function, to be constructed by the following procedure. If the choice function of a court c has been constructed according to that procedure we will denote it by $\mathbb{C}_c^{ts}$, for *time-specific* choice function. The reason for referring to this as the time-specific choice function is that it makes choices of contracts based on constraints which specify for each time period the number of contracts that can be held. For any set of available contracts Y let the choice of court c from this set, $\mathbb{C}_c^{ts}(Y)$, be given by applying the following steps:

- **0:** Reject all contracts $y \in Y$ with $y_C \neq c$.

- **1:** Consider contracts $y \in Y$ with $y_T = 1$. Accept one by one contracts of the highest priority lawyers according to $\succ_c$ until $q_{c,1}$ contracts have been accepted. Once $q_{c,1}$ contracts have been accepted, reject all other contracts y with $y_T = 1$. If a contract of lawyer y_I has been accepted, reject all other contracts y' with $y'_I = y_I$. If there are still contracts left, move to the next step.

- **T≥t≥2:** Consider contracts $y \in Y$ with $y_T = t$. Accept one by one contracts of the highest priority lawyers according to $\succ_c$ until $q_{c,t}$ contracts have been accepted. If a contract of lawyer y_I has been accepted, reject all other contracts y' with $y'_I = y_I$. Once $q_{c,t}$ contracts have been accepted, reject all other contracts y with $y_T = t$. If there are no contracts which have not yet been considered, end the algorithm. Otherwise move to the next step, unless $t = T$, then reject all remaining contracts.

We can now state Lemma 2:

Lemma 2. *The time-specific choice functions satisfy bilateral substitutes and IRC.*

We however have a negative result for the time-specific choice function: it does neither satisfy the unilateral substitutes nor the law of aggregate demand condition, which is illustrated in the following examples.

Example 2. Let $T = \{1, 2\}$, $Y = \{(j, c, 2)\}$ and $x = (j, c, 1)$, $z = (i, c, 2)$. Furthermore let $j \succ_c i$ and $q_{c,1} = q_{c,2} = 1$.

Then we have under a time-specific choice function that $z \notin \mathbb{C}_c^{ts}(Y \cup \{z\}) = \{(j, c, 2)\}$. However we have $z \in \mathbb{C}_c^{ts}(Y \cup \{x, z\}) = \{(j, c, 1), (i, c, 2)\}$, which contradicts unilateral substitutes.

Example 3. Let $Y = \{(i, c, 1), (j, c, 2)\}$, $j \succ_c i$ and $q_{c,1} = q_{c,2} = 1$. Then we have $\mathbb{C}_c^{ts}(Y) = \{(i, c, 1), (j, c, 2)\}$) but we also have $\mathbb{C}_c^{ts}(Y \cup \{(j, c, 1)\}) = \{(j, c, 1)\}$. Hence adding the contract $(j, c, 1)$ to the set of contracts Y reduces the total number of contracts chosen[5].

The unilateral substitutes as well as the law of aggregate demand condition is used by Hatfield and Kojima (2010) and Aygün and Sönmez (2012) to prove (group) strategy-proofness and the rural hospitals theorem for the cumulative offer process. The unilateral substitutes condition is also used to show the existence of a doctor-optimal stable matching. Nevertheless we are able to show that despite of the failure of the unilateral substitutes condition, this result continues to hold in our model. The key to this result is to assume that the preferences of lawyers satisfy the weak impatience property. With that property a situation such as the one in the example above cannot arise. There we had that a contract of lawyer j for a late period was available without contracts of the same lawyer for all earlier time periods being available. Adding one of these earlier time periods then caused lawyer i to be accepted when i was previously rejected. If lawyers however propose early contracts before later ones, such a situation cannot arise in the cumulative offer process.

The time-specific choice function can be seen as a special case of the slot-specific choice function of Kominers and Sönmez (2013), in which a slot at a court corresponds to a specific time and has a number of $q_{c,t}$ copies. Notice that the precedence order among slots is irrelevant when comparing slots at a court for the same time period. For slots of a different time period the implicit precedence order is such that slots corresponding to an earlier time period have precedence over slots of a later time period. Furthermore another difference is that the slot-specific choice function generally allows all slots to accept any contract, while in our case a slot for time period t may only accept contracts which specify this time period. Last, notice that we specify each time-specific slot to use the same ranking over lawyers, while the slot-specific

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[5]We thank Christian Basteck for this example and for correcting a previously incorrect lemma.

choice functions of Kominers and Sönmez (2013) allow slots to have different rankings over lawyers. We could allow for different priorities in different time periods for some of the results. However, as the underlying problem gives us no guidance as to how priorities might differ across time, we chose not to do so. We will later make use of these facts for some of our results.

In the allocation of lawyers to positions there are two further desirable characteristics of mechanisms to achieve: respect of improvements and truthfully reporting special social relations to some particular town. Since the latter simply amounts to making a report which will increase a lawyer's priority at some specific court, assuring reporting such a social relation is akin to the mechanism respecting improvements. The paper Sönmez (2013) uses a property of choice functions, called fairness to prove respect of improvements. Fairness in Sönmez (2013) is defined as follows:

Definition. [Fairness] For any court c, choice function $\mathbb{C}_c$ is **fair** if for any set of contracts $Y \subseteq X$, and any pair of contracts $x, y \in Y$ with $x_C = y_C = c$, $y_I \succ_c x_I$, $y_T = x_T$ and $x \in \mathbb{C}_c(Y)$, then there exists $z \in \mathbb{C}_c(Y)$ such that $z_I = y_I$.

We then have the following lemma 3:

Lemma 3. *The time-specific choice function $\mathbb{C}_c^{ts}$ is fair.*

In addition to the previously used properties of early filling, we now define stability, the central concept of the two-sided matching literature since Gale and Shapley (1962).

Definition. [Stability] An allocation $Y \subseteq \tilde{X}$ is **stable** with respect to choice functions $(\mathbb{C}_c)_{c=1}^{|C|}$ if we have:

1. individually rational: $\mathbb{C}_i(Y) = Y(i)$ for all $i \in I$ and $\mathbb{C}_c = Y(c)$ for all $c \in C$; and

2. there is no court $c \in C$ and a blocking set $Y' \neq \mathbb{C}_c(Y)$ such that $Y' = \mathbb{C}_c(Y \cup Y')$ and $Y' R_i Y$ for all $i \in Y'_I$.

Hence an allocation is stable if each lawyer prefers the assignment to being allocated no contract, each court chooses its assignment over some subset of that assignment and there is no set of contracts such that a court would rather choose that set of contract, the blocking set, when this and the allocation are available, such that the lawyers having contracts in the blocking set weakly prefer those contracts over their assignment. An allocation $Y \subseteq \tilde{X}$ is the **lawyer-optimal stable allocation** if every lawyer weakly prefers it to any other stable allocation.

6.2 Cumulative Offer Process

We now introduce the **cumulative offer process** as defined in Hatfield and Kojima (2010), which is a generalisation of the deferred-acceptance algorithm of Gale and Shapley (1962).

The cumulative offer process takes as input the (reported) preferences of the lawyers as well as the choice function of each court.

- **1:** One (arbitrarily chosen) lawyer offers her first choice contract x_1. The court that is offered the contract, $c_1 = (x_1)_C$, holds the contract if it is acceptable and rejects it

otherwise. Let $A_{c_1}(1) = \{x_1\}$, and $A_c(1) = \emptyset$ for all $c \neq c_1$.

In general,

- **k $\geq$2:** One of the lawyers for whom no contract is currently held by any court offers the most preferred contract, say x_k, that has not been rejected in previous steps. Let $c_k = (x_k)_C$ hold $\mathbb{C}_c(A_{c_k}(k-1) \cup \{x_k\})$ and reject all other contracts. Let $A_{c_k}(k) = A_{c_k}(k-1) \cup \{x_k\}$ and $A_c(k) = A_c(k-1)$ for all $c \neq c_k$.

Now we apply Theorem 1 of Hatfield and Kojima (2010) to show that the cumulative offer process, as just described, in conjunction with the dynamically responsive choice function produces a stable allocation.

Theorem 1. *[Hatfield and Kojima (2010)] Suppose the choice functions of the court used in the cumulative offer process satisfy bilateral substitutes. Then the cumulative offer process produces a stable allocation.*

The existence of a stable matching is the minimum requirement that we ask of an algorithm. By the above result and the fact that the time-specific choice functions satisfy bilateral substitutes, using the time-specific choice functions when running the COP yields a stable allocation. Hatfield and Kojima (2010) further show that if one strengthens the assumptions to unilateral substitutes for the choice functions used, then one can show that the cumulative offer process produces the lawyer-optimal stable allocation. In our case however the time-specific choice functions do not satisfy unilateral substitutes.

Nevertheless one can adapt Theorem 4 of Hatfield and Kojima (2010), as modified by Aygün and Sönmez (2012), which is used in Theorem 5 of Hatfield and Kojima (2010) to show the existence of a lawyer-optimal stable allocation (doctor-optimal in their terminology). To do so, it is sufficient to make an assumption on the preferences of the lawyers, rather than on the choice functions used by the courts. Namely we will assume that lawyers are weakly impatient. Previous results in the matching with contracts literature usually proceeded by restricting the choice functions used by the side of the market which could accept multiple contracts to obtain results, while placing essentially no restrictions on the other side of the market. Here we depart from this approach and relax the restrictions placed on the choice functions used by the side of the market which can accept several contracts and instead put some restrictions on the single-contract side of the market. Both approaches, as we will see, lead to similar results.

Lemma 4. *A contract z that is rejected by a court c at any step of the cumulative offer process using the time-specific choice function $\mathbb{C}_c^{static}$, cannot be held by court c in any subsequent step.*

The key to our proof of this result lies in the specific choice function that we use. This causes lawyers, when a contract of theirs is rejected, to either propose to a new court or to propose to some court at which the lawyer was previously rejected. So if some court c has multiple offers, say z and z' of some lawyer i and holds z, then it will, when receiving a new contract offer from some other lawyer j, never reject z while simultaneously accepting z'. In the proof we heavily rely on Aygün and Sönmez (2012).

With this result in hand, we can now state the following lemma:

Lemma 5. *Suppose lawyer preferences satisfy weak impatience. The outcome of the cumulative offer process using the time-specific choice function $\mathbb{C}_c^{ts}$ produces the lawyer-optimal stable allocation.*

The proof is essentially the same proof as the one of the corresponding Theorem 5 in Hatfield and Kojima (2010) and Aygün and Sönmez (2012). Assuming weak impatience again allows us to relax the unilateral substitutes assumption and instead use the time-specific choice functions which only satisfy the bilateral substitutes assumption. The reason that this works is that because of weak impatience, any sets of available contracts that the courts will have to make choices from are sets such that if a contract x of some lawyer i is available for period t, then contracts for any earlier and feasible period for that lawyer i will also be available. On this restricted domain of sets of available contracts, unilateral substitutes essentially holds for the time-specific choice function, allowing the proofs by Hatfield and Kojima (2010) to go through, with some modifications. Note that we only needed to make use of the assumption of weak impatience for proving Lemma 5.

The result in Lemma 5 is a new result, which is not implied by any of the results in Kominers and Sönmez (2013), since they consider more general slot-specific choice functions than we do here. For general slot-specific choice functions a lawyer-optimal stable allocation is not guaranteed to exist and even when such an allocation exists, the COP is not guaranteed to find it. Lemma 5 above shows that under weak impatience, a lawyer-optimal stable allocation is guaranteed to exist and that it is found by the COP. The following example shows that without weak impatience, the existence of a lawyer-optimal stable allocation is no longer guaranteed.

Example 4. Let i and j prefer $(c, 2)$ to $(c, 1)$ and assume $i \succ j$ with $q_{c,1} = q_{c,2} = 1$. Then the allocation $Y = \{(i, c, 1), (j, c, 2)\}$ is stable, while the COP produced the allocation $Y' = \{(i, c, 2), (j, c, 1)\}$. Notice that j prefers Y, while i prefers Y'.

6.3 Properties of the time-specific stable mechanism

The mechanism, ψ^{ts} is defined to be that mechanism which associates with each preference profile the outcome of the COP using the time-specific choice functions. We will refer to this mechanism as the time-specific stable mechanism. We have the following result:

Proposition 5. *The time-specific stable mechanism is stable and (group) strategy-proof. If in addition lawyers' preferences satisfy weak impatience, then the time-specific stable mechanism is lawyer-optimal stable.*

The first part of the above result follows from Theorem 4 in Kominers and Sönmez (2013), while the second part follows from applying Lemma 5. Instead of applying Theorem 4 in Kominers and Sönmez (2013), an alternative way of obtaining the first part of the above results when lawyers' preferences are weakly impatient is to adapt results in Hatfield and Kojima (2009) making use of the fact that under weak impatience, a lawyer-optimal stable allocation is guaranteed to exist.

One of the problems in the current procedure is that lawyers may be worse off by improving their ranking, for example by obtaining a better grade or having waited longer. The next

proposition shows that this is not the case for the cumulative offer process using the time-specific choice function.

Proposition 6. *The time-specific stable mechanism respects improvements.*

The intuition behind the proof of this result, which is simply an application of Theorem 5 in Kominers and Sönmez (2013), is as follows. Let $\succ_1$ be an unambiguous improvement over $\succ_2$ for lawyer i and let ψ_1 be the associated mechanism. Similarly for $\succ_2$. Suppose the COP were run initially excluding lawyer i under $\succ_1$, which will lead to some allocation X^1. After this, lawyer i proposes contracts in order of preference. This process will terminate for some contract offer x^k, which is i's assignment under the mechanism ψ_1. Running the algorithm under $\succ_2$ without lawyer i will lead to the same initial allocation X^1 since only the ranking of lawyer i has changed. Letting i propose contracts however will lead to the same rejections occurring since $\succ_1$ is an unambiguous improvement over $\succ_2$ until x^k is offered by i, which by assumption is the final allocation under $\succ_1$ but which may nevertheless be rejected under $\succ_2$. From this it follows that i cannot do worse under $\succ_1$ than under $\succ_2$.

This is an important result since it implies that targeted efforts to improve the allocation obtained by specific lawyers through an improvement of their ranking can never hurt these lawyers who those efforts are intended to help. One implication is that when the ranking depends positively on grades, then lawyers are rewarded for better grades by an improvement in their assignment.

The fact that the time-specific stable mechanism respects improvements has a further implication in our application. Lawyers, in the current system, may report to have a special social relationship to a court. Consider now a game which first asks lawyers to report any such information. In a second stage, the priorities of each court would be adjusted to reflect those reports, in case the information lawyers have reported has been verified. In case lawyers do have special social relationship to a court, but do not report it, the choice function remains unaffected. Then we have the following result, which follows by noting that reporting this information leads to an unambiguous improvement in the priority of a lawyer at a court. Since the time-specific stable mechanism respects improvements, reporting this information, holding the strategies of everyone else fixed, cannot make a lawyer worse off, but may lead to an improvement. Hence the following corollary is obtained:

Corollary. *Each lawyer has an incentive to report verifiable information increasing her priority at a court under the time-specific stable mechanism.*

Another desirable property that the time-specific stable mechanism satisfies is fairness.

Proposition 7. *The time-specific stable mechanism is fair.*

To see that the time-specific stable mechanism is fair, let $x, y \in Y \subseteq \tilde{X}$ be two contracts obtained by the time-specific stable mechanism such that $x_I \neq y_I$ and $(x_C, x_T)P_{y_I}(y_C, y_T)$. Then since by the cumulative offer process y_I must have offered (y_I, x_C, x_T) at some step during the process, it must have been rejected. But the only way that (y_I, x_C, x_T) had been rejected while x was accepted is when $x_I \succ_{x_C} y_I$, which implies that the time-specific stable mechanism is fair.

The time-specific mechanism proposed in this section however does not fill positions early. The reason is an inherent conflict between (overall) stability and early filling. To see this consider the following example.

Example 5. There are two periods $t = 1, 2$ and two courts $c = a, b$, each with one position in each period. In the first period, there are two lawyers $I = \{j, k\}$, with common preferences for each $i \in I$: $(a, 1)P_i(a, 2)P_i(b, 1)P_i(b, 2)$. Both courts have priorities such that lawyer $i \succ_c j$. The outcome of the cumulative offer process using the time-specific choice function results in the allocation $\{(i, a, 1), (j, a, 2)\}$, which leaves the position in period 1 at court b unoccupied even though lawyer j is given a position in period 2, thereby violating early filling.

The example shows that (lawyer-optimal) stable outcomes might not satisfy the early filling properties. There is a trade-off between preferences of lawyers from different time periods: On the one hand, a mechanism finding a stable outcome over lawyers from all periods might not fill positions early and by this make future lawyers worse off. On the other hand, guaranteeing early filling could benefit future lawyers at the costs of earlier lawyers, but would not be stable and might violate other desirable properties, such as strategy-proofness.

6.4   Flexible Choice Function

The discussion in the previous sections assumed that each court c could only accept $q_{c,t}$ lawyers in time period t. This assumption was made because the number of positions at each court is determined by the budget of the *Land* several periods into the future so that the courts cannot flexibly set their own capacity for each period. In this subsection we consider the possibility of allowing each court to flexibly determine how to allocate total capacity, which is assumed to be fixed, over several periods. Hence, we no longer have a time-specific capacity constraint but instead have for each court a global constraint on the total number of lawyers that can be accepted.

The set-up remains as before, except that we replace the matrix of court capacities by a vector q of court capacities such that $q_c = \sum_{t \in T} q_{c,t}$. We further introduce the flexible choice function of court c, denoted by $\mathbb{C}_c^{flex}$, to simply select for each offered set X' up to q_c contracts of the highest-priority agents. Whenever an agent has several contracts in the offered set X', the contract with the lowest time is chosen. Notice that the flexible choice function can also be interpreted as a slot-specific choice function in which each slot has precisely the same priority ranking over contracts and there are q_c slots.

We made use of the notions of bilateral and unilateral substitutes earlier. The flexible choice function however satisfies an even stronger condition, the substitutes condition, which is defined formally below:

Definition. [Substitutes] Contracts are **substitutes** for court c if there do not exist contracts $x, z \in X$ and a set of contracts $Y \subseteq X$ such that $z \notin \mathbb{C}_c(Y \cup \{z\})$ and $z \in \mathbb{C}_c(Y \cup \{x, z\})$.

Notice that substitutes implies unilateral substitutes which itself implies bilateral substitutes. The flexible choice function satisfies both the substitutes condition and the law of aggregate demand.

Lemma 6. *The flexible choice function satisfies substitutes and the law of aggregate demand.*

We will call the mechanism ψ^{flex} the flexible lawyer-optimal stable mechanism (FLOSM) that associates to each profile of preferences the outcome of the cumulative offer process when using the flexible choice function for each court. We may then apply the results in Hatfield and Kojima (2010) to obtain the result that the FLOSM produces a matching which is the lawyer-optimal stable mechanism independent of whether lawyers' preferences satisfy weak impatience. Furthermore, we may apply the results of Kominers and Sönmez (2013) that were used earlier to conclude that FLOSM is (group) strategy-proof, respects improvements and is fair. To save space, we omit the associated propositions and their proofs.

We focus in this subsection on comparing the time-specific stable mechanism to the FLOSM. Since the FLOSM essentially removes a restriction on capacity in specific time periods and replaces it by an aggregate constraint, one might conclude that the FLOSM should Pareto-dominate the time-specific stable mechanism. However, as we will show below, this is not true. The following example demonstrates that there are instances in which no welfare comparison can be made between the time-specific stable mechanism and the FLOSM.

Example 6. $I = \{i, j, k\}$, $C = \{a, b\}$ with $q_{a,1} = q_{b,1} = q_{a,2} = 1$ and $i \succ j \succ k$ for both courts and preferences are given by:

$$i: \qquad (a,1)P_i \emptyset$$
$$j: \quad (a,1)P_j(b,1)P_j(a,2)$$
$$k: \quad a,1)P_k(a,2)P_k(b,1)$$

The outcome of the time-specific stable mechanism is $\{(i,a,1),(j,b,1),(k,a,2)\}$. The outcome of the FLOSM is $\{(i,a,1),(j,a,1),(k,b,1)\}$. Notice that while j is better off under FLOSM, since j now obtains a place at a in period 1, we have that k is now worse off. Therefore, neither of the two mechanisms dominates the other.

Example 7. $I = \{i, j, k\}$, there is only one court, with $q_{c,1} = q_{c,2} = q_{c,3} = 1$ with priorities $i \succ j \succ k$ and each lawyer is impatient. Then FLOSM yields $\{(i,c,1),(j,c,1),(k,c,1)\}$, while the time-specific stable mechanism yields $\{(i,c,1),(j,c,2),(k,c,3)\}$. Hence both j and k are better off under FLOSM than under the time-specific stable mechanism.

Example 8. $I = \{i, j, k, l\}$ there are two courts, with $q_{c,t} = 1$ for each court $c = a, b$ and each period $t = 1, 2$ except for $q_{b,2} = 0$. Priorities for court a are $i \succ_a k \succ_a l \succ_a j$, while priorities for court b are $j \succ_b k \succ_b i \succ_b l$.

$$i: \qquad (a,1)P_i \emptyset$$
$$j: \qquad (a,2)P_j(b,1)P_j \emptyset$$
$$k: \quad (b,1)P_k(a,1)P_k(a,2)$$
$$l: \qquad (a,1)P_l \emptyset$$

The outcome of the time-specific stable mechanism is given by $\{(i,a,1),(j,a,2),(k,b,1),(l,\emptyset)\}$, while the outcome of the FLOSM is given by $\{(i,a,1),(j,b,1),(k,a,1),(l,\emptyset)\}$. Hence both j and k are worse off under FLOSM than under the time-specific stable mechanism.

There are two differences between FLOSM and the time-specific stable mechanism which work to off-set each other. First, under FLOSM the set of feasible contracts is larger than under

the time-specific stable mechanism. This is because more lawyers can now be allocated to a court at a specific time. Second, under FLOSM there can be more blocking coalitions than under the time-specific stable mechanism.

We are however able to show that FLOSM is not Pareto dominated by the time-specific stable mechanism under an additional assumption on the priorities used by the courts: the acyclicality condition of Ergin (2002). To use this condition we will need to introduce some additional notation. For all $c \in C$ and for all $i \in I$ let $U_c(i) = \{j \in I | j \succ_c i\}$. Then acyclicality is defined as follows.

Definition. [Acyclicality] Let $\succeq$ be a priority structure and q a vector of quotas. A cycle is constituted of distinct $a, b \in C$ and $i, j, k \in I$ such that the following are satisfied:

- Cycle condition (C): $i \succ_a j \succ_a k \succ_b i$
- Scarcity condition (S): There exist (possibly empty) disjoint sets of agents
  $I_a, I_b \subset I \setminus \{i, j, k\}$ such that $I_a \subset U_a(j)$, $I_b \subset U_b(i)$, $|I_a| = q_a - 1$
  and $|I_b| = q_b - 1$

A priority structure is acyclical if it has no cycles.

Given the above definition of cyclical priority structures, we can immediately apply Theorem 1 in Ergin (2002) to obtain the result that when priorities (which are the same for FLOSM and the time-specific stable mechanism) satisfy acyclicality, then FLOSM is never Pareto dominated by the time-specific stable mechanism.

Proposition 8. *Suppose priorities are acyclic. Then for given reported preference profiles the time-specific stable mechanism never Pareto dominates the FLOSM.*

Proof. Notice first of all, that FLOSM is identical to a mechanism in which agents simply report their preferences over courts, which are given by their preferences over court and time, by only considering the most-preferred contract for each court and the lawyer-optimal deferred-acceptance algorithm is used. We can then apply Theorem 1 in Ergin (2002) which states that the outcome of the lawyer-optimal stable mechanism is Pareto efficient if and only if priorities are acyclic. From this it follows that FLOSM is Pareto efficient. Note further that the set of feasible contracts under the time-specific capacity constraints is smaller than under the aggregate constraints. Therefore FLOSM cannot be Pareto dominated by the time-specific stable mechanism. □

Under acyclical priorities we cannot show that the time-specific stable mechanism is not Pareto dominated by the FLOSM. The reason is that, while under acyclical priorities and time-specific capacity constraints, the time-specific stable mechanism is Pareto efficient with respect to the time-specific constraints. However, the FLOSM does not face these restrictions and hence may yield outcomes which are not feasible under those constraints. Hence there are cases in which FLOSM Pareto dominates the time-specific stable mechanism under acyclic priorities. To see this, change Example 7 by adding another court and another lawyer while extending the preferences and priorities of existing lawyers and the existing court by adding the new court and lawyer to the end of the preference and priority list respectively. The new court and the new lawyer both have the same priorities and preferences respectively as the existing agents. Then

it is easy to see that FLOSM leads to an outcome that Pareto dominates the outcome of the time-specific stable mechanism.

Acyclical priorities may obtain in our application whenever each court bases their priorities solely on the average grade of the lawyers and their arrival time in a symmetric way. We then have homogeneous preferences among courts, implying that there are no cycles. However when courts differ in the way that they construct their priorities, by for example giving higher priority for lawyers with a special social relationship, then this may result in cycles being present.

The FLOSM can also be shown to not be Pareto dominated by the time-specific stable mechanism when all lawyers have homogeneous preferences irrespective of the acyclicality condition. The reason is that, essentially, both mechanisms will be Pareto efficient given their constraints. However the constraints under FLOSM are less severe, implying a greater set of feasible allocations. Hence the time-specific stable mechanism cannot Pareto dominate FLOSM under homogeneous lawyer preferences. We thus state the following proposition, without an explicit proof.

Proposition 9. *Suppose lawyer preferences are homogeneous. Then FLOSM is never Pareto dominated by the time-specific stable mechanism.*

6.5 Slot-specific Mechanism

The Berlin mechanism constructs the set of lawyers to be matched by using quotas for different types of students. For example, in Berlin the first 20% of spaces are reserved for lawyers with a grade above 10, with the next 10% of spaces reserved for special hardship cases. The remaining spaces are allocated by waiting time, of these 80% have higher priority for lawyers graduating from a Berlin-based university. Grades are used as tie-breaker. This situation lends itself to an analysis similar to the one used for affirmative action programs in the United States. One could change the time-specific and the flexible choice functions that were introduced earlier to take account of these affirmative action concerns. The mechanism based on these choice functions can then be analysed using the set-up in Kominers and Sönmez (2013) whereby some slots have higher priority for some types of lawyers. For example, each court could have a predetermined number of "waiting-time" slots and "social hardship" slots in each period. Here provisionally assigned lawyers could automatically remain in highest priority at waiting time slots in future periods. As we are focusing on the time of allocation as a novel contract term, we leave it at these observations.

7 Conclusion

We study the centralized allocation of lawyers to their mandatory post-graduate traineeship at regional courts in Germany. In this market not all applying lawyers can be matched to a position at once, since in some periods the number of applicants exceeds the number of open positions at courts. Then some lawyers have to wait to be assigned a position. This is a new application of the matching with contracts model in which the time of allocation takes the role of a contract term. This allows us to employ the matching with contracts model pioneered by Hatfield and

Milgrom (2005) and set up a lawyer-court matching problem, where a contract consists of a lawyer, a court and the time period of employment start.

First, we analyse the currently used matching procedure, which we call Berlin Mechanism, and demonstrate it to be unfair and that it does not respect improvements. However, the Berlin Mechanism satisfies early filling, as earlier feasible positions never remain open, while available lawyers receive a later assignment.

Second, we propose a new stable mechanism for which we construct choice functions out of preferences of lawyers and priorities of courts. Although our choice function fails the important unilateral substitutes condition that was needed in the proofs for many results in the literature, we are able to obtain similar results, which we consider an interesting theoretical contribution. For this we only need a mild assumption on the preferences of lawyers: weak impatience. This ensures that lawyers will always prefer earlier to later assignments at the same court. With this in hand, we are able to apply previous theorems from Hatfield and Kojima (2010) and Aygün and Sönmez (2012) and show that when all future lawyer arrivals and reported preference lists are known upfront, then the time-specific repeated cumulative offer process based on our choice functions produces the lawyer-optimal stable allocation. This mechanism satisfies fairness, respect of improvements as well as strategy-proofness, however it might not fill positions early. It should be noted that even without the weak impatience assumption we can demonstrate (group) strategy-proofness and improvement respecting by directly applying the results of Kominers and Sönmez (2013), who provide a more general model allowing for slot-specific priorities. However, our path through weak impatience goes beyond their results as it also guarantees lawyer-optimality. It might appear that under weak impatience the time of allocation plays a similar role as money, in the sense that an earlier allocation is preferred similar to higher wages being preferred. However this resemblance does not extend to the choice function used by the courts. Whenever a court has to choose from two contracts of the same lawyer, it would choose the one with the earlier time period. In a model with money, this would be akin to the employer preferring workers at higher wages.

Third, we propose another lawyer-optimal stable mechanism, which allows for a more flexible allocation of court positions over time. If courts face only aggregate budget and thus capacity constraints, then it might be worthwhile implementing this flexible lawyer-optimal stable mechanism (FLOSM). Although we find that in general welfare comparisons between our time-specific stable mechanism and FLOSM are not possible, we can show that under acyclicality of court priorities FLOSM will never be dominated by the former. The same result is true for homogeneous lawyer preferences. What is more, we could also directly apply our mechanism to the cadet-branch matching problem. In this problem there is only one aggregate constraint on capacities, while in our lawyer-court matching problem we have more specific court-time constraints. Therefore, even without an adapted weak impatience assumption, our choice function would satisfy unilateral substitutes and the mechanism would return a cadet-optimal stable allocation.

Fourth, we can apply the slot-specific matching with contracts model by Kominers and Sönmez (2013) to our setting. For this we assume different court slots to exhibit different priorities, for example by either ranking waiting time or grades first. This mirrors some elements of the currently used quota system.

We proposed two mechanisms to be used in allocating lawyers to courts in Germany. We note here that both mechanisms would require lawyers to state their full preferences, that is preferences over courts and time. In practice eliciting such preferences may be infeasible as the preference space is too large. For practical recommendation one would therefore need to facilitate reporting preferences using a minimal amount of information, by for example restricting preferences to satisfy weak impatience and/or time independence. To provide better guidance on how to implement the proposed mechanisms in practice one would need further information on the likely intensity of the preferences of the lawyers. However, given that this is a high-stakes decision faced by the lawyers we expect it to be reasonable to ask for fairly detailed preferences in applying the mechanisms.

An interesting extension of our model would be to consider how our proposed mechanism behaves when it needs to be applied for each period over a number of periods. Dur and Kesten (2014) consider a problem in which a set of students is to be matched to colleges, but in which the set of colleges is partitioned. They show that when the assignment happens sequentially, it is inherently difficult to have a mechanism be non-wasteful, strategy-proof, fair and to respect improvements. Such results would also apply in a dynamic version of our model in which the time-specific lawyer-optimal mechanism were applied repeatedly. A related problem, that we have ignored so far, is how to manage capacity. While we assumed that capacities were given exogenously, in a dynamic procedure with excess demand one may not make available some capacity at some courts to ensure that future agents are not unduly disadvantaged by earlier agents taking these positions. Future research should address this question.

Finally, having analysed the operation of the currently used procedure (Berlin Mechanism) and that of possible alternatives for the future, it would be helpful to apply these algorithms in practice. It would be worthwhile to understand whether in practice this has a detrimental effect on the resulting allocations relative to the currently used procedure and whether the proposed mechanisms improve upon the Berlin Mechanism.

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Appendix

Proof. [**Proposition 2**] First we show that the matching resulting from the Berlin Mechanism employing the lawyer-proposing GS-DA procedure is fair. To do so, suppose otherwise. The second step of the procedure employs the fair GS-DA mechanism and therefore is fair by construction. Therefore, whenever there exist opportunities for unfairness, these must involve agents being allocated positions in different time periods, i.e. in the first step. Then there exist a lawyer who obtains a position at some court at a later period, although due to his strict impatience he would have preferred the earlier position. If his priority was lower than any other lawyer's who was allocated in the earlier period, the allocation would not be unfair. Therefore it must be that he has a higher priority than some other lawyer in the earlier period. But then he would have already been chosen by the court-side in the first-step of the first period, a contradiction.

Second, we show that the Berlin Mechanism respects improvements. To do so, suppose otherwise. Then there exists a lawyer who obtains a strictly worse position after his grades have improved. First consider an improvement in grades which leaves the time period of allocation unchanged. But then in the second step his allocation can only (weakly) improve since the GS-DA mechanism respects improvements. Hence the change in grades must change the time period of allocation. However an improved grade can only reduce the waiting time of an agent. Since each agent's preferences satisfy strong impatience, no agent can be made worse off by this change. This contradicts the assumption that the procedure does not respect improvements. $\square$

Proof. [**Lemma 2**] It should be clear from the description of the time-specific choice functions satisfy IRC. What remains to be verified is bilateral substitutes.

Suppose to the contrary that there exist contracts $x, z \in X$ and $Y \subset X$ such that $x_I, z_I \notin Y_I$ and $z \notin \mathbb{C}_c^{ts}(Y \cup \{z\})$ but $z \in \mathbb{C}_c^{ts}(Y \cup \{x, z\})$. Note first that z cannot have been rejected because another contract with z_I has been accepted in any stage. Hence contract z must have been rejected when considering contracts from period $s = z_T$ and because z_I had a lower priority according to $\succ_c$ than all other contracts accepted for period s. If $z_T = x_T$ then since contracts are chosen in order of priority, this cannot mean z is accepted when x is also available. But also when $x_T \neq z_T$ this cannot imply that z will be chosen. If $x_T > z_T$, then the presence of x does not affect the choice of z since the latter is considered before the former. If $x_T < z_T$ then if the priority of x_I is low, x will be rejected and not affect the choice of z. If the priority of x_I is high, then it may be chosen. This can lead some other contract y to be rejected. If there are other contracts y' in Y such that $y_I = y'_I$ then this may lead to some more contracts to be considered in z_T but this cannot lead to z being chosen. $\square$

Proof. [**Lemma 3**] Suppose to the contrary that for court c and a set of contracts Y with elements $y, x \in Y$ such that $y_C = x_C = c$, $y_I \succ_c x_I$, $y_T = x_T$ and $x \in \mathbb{C}_c^{ts}(Y)$ but that there does not exist $z \in \mathbb{C}_c^{ts}(Y)$ with $z_I = y_I$. Note that in particular this implies that $y \notin \mathbb{C}_c^{ts}(Y)$. Then since such a z does not exist, it must be that in step $y_T = t$ of the procedure to construct $\mathbb{C}_c^{ts}$, y has not yet been rejected. So in step t both x and y are still available. Now x is accepted in step t since $x \in \mathbb{C}_c^{ts}(Y)$ while y is rejected, since $y \notin \mathbb{C}_c^{ts}(Y)$. This contradicts $y_I \succ_c x_I$, since the procedure to construct the time-specific choice function would have selected the contract of

the agent with the better ranking. □

Proof. [**Proposition 1**] The time-specific choice functions $\mathbb{C}_c^{ts}$ satisfy bilateral substitutes, so we can apply theorem 1 of Hatfield and Kojima (2010) □

Proof. [**Lemma 4**] Towards a contradiction let k' be the first step a court c holds a contract z that was previously rejected at step $k < k'$. As z is rejected at step k, it was on hold by court c at step $(k-1)$ or it was offered to court c at step k. In either case no other contract of lawyer z_I could be on hold by court c at step $(k-1)$. But then, since z is the first contract to be held after an earlier rejection, court c cannot have held another contract by lawyer z_I at step k. That is $z_I \notin \left[\mathbb{C}_c^{ts}(A_c(k))\right]_I$. Since z is rejected at step k, this means that for all $x \in \mathbb{C}_c^{ts}(A_c(k))$ with $x_T = z_T$, we must have $x_I \succ_c z_I$. Let $\left[\mathbb{C}_c^{ts}(A_c(k))\right](z_T)$ denote the set of such contracts in time z_T. Given the definition of $\mathbb{C}_c^{ts}$, $z \in \mathbb{C}_c^{ts}(A_c(k'))$ implies that some contract $x \in \left[\mathbb{C}_c^{ts}(A_c(k))\right](z_T)$ can no longer have been under consideration in step t of the procedure to find the court's choice. But for that to have happened, it must be that some contract y with $y_I = x_I$ and $y_T < x_T$ has been accepted in step k'. But this cannot be since by assumption z is the first contract that was rejected and subsequently accepted and because x_I cannot have offered a contract in step k' since a contract of x_I was held by the court in period $k'-1$. Hence a contradiction. □

Proof. [**Lemma 5**] To prove the theorem, it is sufficient to show that for any stable allocation $X' \subseteq \tilde{X}$ and any contract $z \in X'$, contract z is not rejected by the cumulative offer algorithm when the time-specific choice function is used. To obtain a contradiction, suppose not. Let k be the first step where court $c = z_C$ rejects contract z, and let $Y = \mathbb{C}_c^{ts}(A_c(k))$. Then by IRC, $z \notin \mathbb{C}_c^{ts}(Y \cup \{z\})$. Then by lemma 4, $z_I \notin Y_I$. As k is the first step a contract in any stable allocation is rejected, every lawyer in Y_I weakly prefers their contract in Y to their contract in X' which is stable by assumption. We then consider two cases:

- Case 1: $z \notin \mathbb{C}_c^{ts}(Y \cup X')$. In this case, court c blocks allocation X' together with lawyers in Y_I, contradicting stability of X'.

- Case 2: $z \in \mathbb{C}_c^{ts}(Y \cup X')$ But this cannot be, since for any $x \in \mathbb{C}_c^{ts}(Y)$ with $x_T = z_T$, we have for all $s < t$, $(x_I, c, s) \in Y$ by weak impatience. Therefore the addition of contracts cannot result in z being chosen when both Y and X' are available due to the way the time-specific choice function is constructed. A contradiction.

□

Proof. [**Lemma 6**] First, consider for a contradiction some $Y \subset X$ and two contracts $x, z \in X \backslash Y$ such that $z \notin \mathbb{C}_c^{flex}(Y \cup \{z\})$ and $z \in \mathbb{C}_c^{flex}(Y \cup \{x, z\})$. This contradicts the flexible choice function being used. Since z is not chosen when $Y \cup \{z\}$ was available, there are at least q_c contracts of q_c agents that are ranked higher by c than z_I. This fact does not changed when contract x is also available. Therefore the flexible choice function satisfies the substitutes condition.

To show that the law of aggregate demand holds, suppose to the contrary that there is some $Y \subset X$ and some $z \in X \backslash Y$ such that $|\mathbb{C}_c^{flex}(Y)| > |\mathbb{C}_c^{flex}(Y \cup \{z\})|$. This contradicts the flexible choice function being used, since contracts are accepted based on the ranking of the

lawyer mentioned in the contract. Adding a further contract of a new agent can only lead to either an existing agent's contract being replaced by the new contract, no change in the chosen set or an additional contract being chosen. In either case, the number of accepted contracts is unchanged or greater when more contracts are available. If the added contract is from a lawyer who has another contract in the set Y then either the new contract replaces the existing contract or it does not. In either case, the total number of contracts that is chosen does not change. □