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Mediated Audits

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Abstract

I study the optimal audit mechanism when the principal cannot commit to an audit strategy. Invoking a revelation principle, the agent reports her type to a mediator who assigns contracts and recommends the principal whether to audit. For each reported type the mediator randomizes over a base-contract and the audit contract, accompanied by a recommendation to audit. For large penalties the optimal mechanism uses strictly more contracts than types and cannot be implemented via offering a menu of contracts. The analysis provides a proper benchmark for studying auditing under limited commitment and sheds new light on the usefulness of mediation in contracting and on the design of optimal mechanisms.

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Keywords: Auditing, limited commitment, mediation, contract theory

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1 Introduction

For more than two decades, the literature on contracting with audits discusses the commitment problem implied by these contracts.[1] On the one hand, auditing allows the principal to learn the agent's private information, and thereby relaxes the incentive problem.[2] But on the other hand, an optimal contract already elicits all the private information. Ex-post it is therefore not in the principal's interest to carry out a costly audit. Forcing the principal to audit through contractual means is also difficult. In particular, if the optimal audit policy is random, it is extremely difficult to monitor whether the principal is adhering to the contract.

Addressing this issue is not straightforward, because the lack of commitment undermines the simple structure of 'menu offers'. Recall that under full commitment, the revelation principle implies the principal can do no better than offering a menu with as many contracts as types, and in such a way that each type prefers the contract designed for her over any other contract in the menu. Moreover, the agent chooses her designated contract with certainty. With limited commitment, the latter is no longer correct: *Provided* the principal offers a menu it may turn out optimal that the agent chooses randomly rather than deterministically.[3] Khalil (1997) applies this idea to study audit contracts under limited commitment. Yet then there is no justification for using menu offers in the first place.[4] After all, a menu offer is a short-cut for asking the agent to report her type to the principal, who can alternatively use more elaborate communication protocols such as multiple rounds of message exchange, communication via interested or disinterested third parties, through noisy channels, etc.

In this article I study audit contracts, without commitment to an audit strategy.[5] In contrast to the existing literature, I do not restrict to menu offers, but rather apply a revelation principle for sequential games. The revelation principle allows me to focus on direct and incentive-compatible mechanisms which employ a mediator: The agent confidentially reports all private information to the mediator, who (randomly) picks a contract and recommends the principal whether to audit. Furthermore, I only have to consider mechanisms, where the agent is truthful and the principal obediently follows the mediator's recommendation.

[1] For early references see, e.g., Bolton and Scharfstein (1990), Hart (1995) and Khalil and Lawarrée (1995).

[2] See for example, Baron (1984), Baron (1989), Baron and Besanko (1984), Border and Sobel (1987), Demski, Sappington and Spiller (1987), Dunne and Loewenstein (1995), Graetz, Reinganum and Wilde (1986), Hart (1995), Kofman and Lawarrée (1993) and Mookherjee and Png (1989).

[3] Bester and Strausz (2001) show that when the agent is to choose from a menu it is w.l.o.g. to offer a menu from the same cardinality as the type space *and* that each agent picks the designated contract with strictly positive probability - but the latter probability may be smaller than one.

[4] Bester and Strausz (2007) provide an example where menu offers are sub-optimal and the principal benefits from noisy communication. In their setting the agent reports to a machine that randomly sends messages to the principal. A similar communication protocol is used in the literature on cheap talk, e.g., Blume, Board and Kawamura (2007).

[5] That is, without commitment to a pre-specified probability of an audit, conditional on observable and verifiable information (such as the agent's report).

However, the revelation principle leaves open the question of the number of different contracts the mechanism uses. When restricting to menu offers the number of contracts does not exceed the number of types.[6] But as I outlined above, menu offers are in general sub-optimal and the maximal number of contracts is not determined. A main contribution of this paper is to show that the optimal mechanism uses at most one additional contract - one more than the number of types. The mediator randomly chooses a contract for each type-report and the randomization is over two contracts. Furthermore, one contract is assigned to all types, i.e., for each type there is a base-contract and then there is an additional contract that can be assigned to all types. Along with the latter contract the principal receives a recommendation to audit. For all other contracts, i.e., those that can be identified with the respective type, no audit is recommended.

Building on these results, characterizing the optimal mechanism is a straightforward optimization problem. Two main findings are: First, there is a unique threshold, such that audits are used if and only if the penalty exceeds the threshold. Second, I show that for sufficiently large penalties the optimal mechanism features *strictly more* contracts than types.

How do audits work under limited commitment? To grasp some intuition, consider first the optimal contract without auditing. In this contract, the agent earns an informational rent as a compensation for not misreporting her type. The threat of an audit allows for reducing rents, via penalizing false reports. Under limited commitment, this is complicated by the need to provide incentives for the principal to actually carry out the audit: An audit is costly, so the expected penalty must at least outweigh its cost. To achieve this, an additional contract is introduced – the *audit contract* – which the mediator allocates to both types with positive probability. The share of types under the audit contract is such that the principal's expected revenue from auditing - via the penalty payments - exceeds the audit cost. Introducing the audit contract has various effects on the principal's profit: First, both types produce differently from what they did before, which can both increase or decrease the principal's profit. Second, it directly affects the agent's rent via the adjusted quantity. And third, it has the deterring effect via the penalty payment, that is applied only to the type with an incentive to misreport. How these effects interact depends on the fundamentals of the problem, but it is immediate that for large levels of the penalty the latter effect dominates and, hence, audits are beneficial.

When penalties are high, the principal is able to reduce all rents to zero. The mechanism now deals with the inefficient quantities, to reduce also this distortion. Because audits have a constant marginal cost, but the marginal benefit from reducing distortions in quality vanishes as the allocation approaches the first-best, it becomes optimal to reduce the frequency of audits for large penalties. A reduced audit frequency implies that no type is audited with certainty and therefore the optimal mechanism features strictly more contracts than types: for each type a base-contract

[6] See Bester and Strausz (2001) for details.

that is type dependent *and* and the audit contract accompanied by the recommendation to audit. I further show that this mechanism cannot be implemented via offering a menu of contracts to the agent. Furthermore, the allocation only approximates the first-best in the limit as penalties become infinite, which stands in contrast to results obtained by Khalil (1997) who restricts contracts to menu offers.[7]

The implications of my results are manifold. First of all, I provide the proper benchmark for studying audits under limited commitment. Several authors have studied more elaborate models of auditing under limited commitment with the aim of explaining particular institutional patterns, such as independent audit agencies. In this vein, Khalil and Lawarrée (2006) derive a demand for external auditing, when the principal cannot commit to an audit policy and on top of that the internal auditor can collude with the agent. Similarly, Melumad and Mookherjee (1989) show that the principal benefits from delegating the authority over auditing to an independent agency of which the principal controls the budget. In both examples, the standing of internal audits is aggravated by restricting the principal to offer simple menus of contracts. Consequently, the insights presented in this paper raise the question to what extent these results are driven by limited commitment itself, or by the restrictive use of sub-optimal mechanisms.

A second implication concerns the structure of the optimal mechanism. The principal's decision whether to audit is solely based on the mediator's recommendation and the observed contract. Confidentiality of the agent's report to the mediator is a crucial feature for the mechanism to work. A possible way of implementation has the principal hiring consultants with a mandate to negotiate the contract. The same consultants then recommend the principal whether to audit.

On a more fundamental level, this paper provides novel insights for the study of optimal contracts under limited commitment, which facilitate our understanding of the beneficial role of mediation in contract theory. The mediated mechanism correlates the agent's reported information with the recommendation to the principal, an aspect that is inherent in the idea of *communication equilibrium*, but has not been explored in contract theory yet. Furthermore, to the best of my knowledge, this is the first paper that transforms a contracting problem with limited commitment via application of a revelation principle into a tractable form.[8] For future research this offers the possibility for applying the concept of mediation to other contexts and for studying the benefits of mediation in general.

The remainder of this paper is organized as follows: Section 2 reviews the related literature and section 3 presents the model. Section 4 provides the preliminary result that mediation is beneficial compared to menu offers. In section 5 I analyze the optimal mechanism under limited

[7] See also Khalil and Lawarrée (1995).

[8] Bester and Strausz (2007) also provide a tractable model of contracting with limited commitment, but they make restrictions on the modes of communication. Consequently, it remains unclear under which conditions they find generally optimal contracts. Furthermore, their Assumption 1 is restrictive, see also footnote 20.

commitment. Section 6 briefly discusses whether the optimal mechanism can be implemented using simple communication protocols, and section 7 concludes. All proofs are relegated to the appendix.

2 Related Literature

The literature on auditing started with Baron and Besanko (1984), who study the problem of regulating a firm under asymmetric information. They characterize the optimal auditing policy when the regulator can commit to an auditing strategy. Khalil and Lawarrée (1995) take up the commitment problem and sketch how shirking by the agent is required to provide incentives for the principal to carry out an audit. A formal analysis of auditing under limited commitment is carried out by Khalil (1997).[9] He restricts the analysis to menu offers, but allows for randomization by the agent. A typical contract in his setting involves a version of the inspection game: The low-cost type (in a two-type model) randomly reports high or low costs and the principal randomly audits after a high-cost report. I argue in this paper, that the random-audit contract is never optimal.

Also the literature on costly-state verification, nicely surveyed by Attar and Campioni (2003), studies the issue of limited commitment to the audit policy. As compared to the articles stated above, the private information arises *after* signing the contract, which substantially alters the effects at work. Dunne and Loewenstein (1995) study a model where several agent's compete for a principal's project. The final cost report, that includes a cost-shock realized after signing the contract, can be audited by the principal. Khalil and Parigi (1998) study loan contracts where the bank can audit in case of default. They argue that the loan-size can be used to reduce under-reporting. Picard (1996) studies optimal insurance contracts when insurers cannot commit to audit strategies and contracts arise in a competitive market. Lang and Wambach (2013) prove that an insurer avoids commitment and strategically chooses ambiguity about audits to fight insurance fraud. All these works restrict the contract space to mechanisms where the agent reports private information to the principal - generals mechanism are not studied.

A strand of the literature cites limited commitment as a reason for the frequently observed separation of auditing. Khalil and Lawarrée (2006) derive a demand for external audits when internal commitment is limited. Melumad and Mookherjee (1989) study delegation in a model of tax compliance. The government can implement the full-commitment solution by delegating authority over the audit policy. Because the limited commitment benchmark uses restrictive assumptions on the contract space, it is not clear whether these results are solely explained by limited commitment

[9] Chatterjee, Morton and Mukherji (2008) study a continuous type problem, where the agent reports the value of the firm. Under-reporting the firm's value implies pocketing the difference. Audit penalties are composed of returning the amount of fraud and a proportional penalty. No general contracts are studied, but only those where the agent reports the firm's value (and may misreport it).

or rather based on restrictive assumptions on feasible mechanisms.

Surprisingly, little is known about optimal contracts under limited commitment in general. Bester and Strausz (2001) provide a revelation principle when communication is limited to one round of face-to-face communication, which is equivalent to offering a menu of contracts. The same authors show that noisy communication can be beneficial, in Bester and Strausz (2007). Their results depend on an assumption about the agent's utility function, which is not satisfied in the model I study, which precludes adopting their solution procedure. Furthermore, the restriction to particular noisy communication protocols leaves open the question of whether the resulting mechanisms are optimal also in general.

The approach to communication used in this paper is borrowed from the game-theoretic literature, e.g., Myerson (1986) and Forges (1986). The fact that multi-stage communication already enhances welfare has been demonstrated by Forges (1990) and Krishna and Morgan (2004). With indirect communication, i.e. via a mediator or a noisy channel, further improvements are possible (see e.g. Myerson (1986) and Forges (1986)).

Recently, mediation has found its way into contract theory. Rahman and Obara (2010) show that mediation can virtually implement first-best effort choices in a team problem where budget-balance is required. Strausz (2012) links this result to general insights from mechanism design, i.e., to Myerson (1982).

The impact of various communication protocols is perhaps best understood in the area of cheap talk. Crawford and Sobel (1982) provide the benchmark with a single round of face-to-face communication, which is extended to multiple rounds by Krishna and Morgan (2004). Mediation is added by Goltsman, Hörner, Pavlov and Squintani (2009). That noisy communication is already sufficient to achieve the outcome under mediation, is shown in Blume et al. (2007).

3 Model

I use a two-type version of the well-known model of Baron and Myerson (1982) and augment it by the principal's ability to audit. There are two risk-neutral players, a principal and an agent. The principal hires the agent to produce some good. The value of q units of the good to the principal is given by the strictly concave and strictly increasing function $V(q)$. Further assume $V(0) = 0$.

The agent has constant marginal cost of production θ and no fixed cost. With prior probability $\phi \in (0, 1)$ marginal costs are low, that is, $\theta = \theta_l > 0$. Costs are high, that is, $\theta = \theta_h > \theta_l$ with probability $1 - \phi$. Further, let $\Delta\theta := \theta_h - \theta_l > 0$ denote the difference in agent's costs. Efficient production levels q_i^o are given by

$$V'(q_i^o) = \theta_i, \quad i = l, h. \tag{1}$$

The marginal cost θ is private information of the agent. The publicly observable output q belongs to the principal, who compensates the agent with a transfer t. When specified by the contract, a punishment $P > 0$ can be applied to the agent, as outlined below. The agent maximizes her expected payoff, equal to $t - \theta q - \mathbb{E}[P]$. The reservation utility of the agent is zero and I assume that it is always optimal for the principal to employ either type of the agent.[10]

In addition, the principal possesses an audit technology, that allows to learn the agent's true costy after production took place.[11] As an example, the principal can send an inspector in order to check the agent's accounts. From investigating the accounts, the inspector can infer total production costs and thereby learn the true parameter θ. Upon conducting an audit the principal incurs a cost of $c > 0$.

As mentioned above, the contract can provide for the application of a punishment to the agent. The level of the punishment is exogenously given by $P > 0$, i.e., the contract either calls for a punishment of P or no punishment at all.[12] This payment is, in particular, independent of the output-based transfer from the principal to the agent.[13] Furthermore, the transfer is paid *before* the principal audits, which precludes linking the transfer to the principal's audit decision.[14]

Together with the auditing there are two stages: First, a production stage in which the agent produces the output q and receives the transfer t. Second, an audit stage in which the principal decides whether to audit, learns the audit results and potentially the punishment is applied to the agent. The outcome of the first period is observable to all parties, in particular the principal observes the output and the transfer payment before deciding whether to audit.

Invoking a revelation principle for sequential games by Myerson (1986), a *mechanism* $\Gamma = \{\pi_l, \pi_h\}$ is a tuple of two probability distributions, where each of the distributions is over contracts and recommendations.[15] A *contract* is a triple $(t, q, P(\cdot|\cdot))$, consisting of a transfer t, a quantity q and a penalty assignment $P(\cdot|\cdot)$. The contract specifies the terms of trade between agent and principal, i.e., the agent is asked to deliver q against a payment of t. It further establishes rules for the application of penalties, via the function $P(\cdot|\cdot)$, as specified below. The recommendation

[10]The Inada conditions $\lim_{q\to 0} V'(q) = +\infty$ and $\lim_{q\to 0} qV'(q) = 0$ are sufficient to rule out shutdown (see Laffont and Martimort, 2009, chap. 2.6).

[11]Pollrich (2015) allows for imperfect audits, see also section 7 for a discussion of this assumption.

[12]Under full commitment this is the maximum punishment principle. Pollrich (2015) examines validity of this principle under limited commitment, see also section 7 for a discussion of this assumption. One possible interpretation is, that P is enforced by a court. Though the contracting parties have all flexibility in determining conditions that are seen as breach, they are committed to penalty payments imposed by jurisdiction.

[13]If the punishment includes repaying the transfer, the first-best is virtually implementable both under full and limited commitment. In this case, the size of the transfer serves as a punishment which is unbounded by nature. This result is similar to Nalebuff and Scharfstein (1987), where the penalty itself is unbounded.

[14]Making the transfer contingent on the audit decision alleviates the commitment problem, as demonstrated by Strausz (2001, Corollary 1). In particular, with perfect audits there is no commitment problem at all.

[15]For reasons of tractability I assume that each π_i has finite support. This assumption substantially simplifies the exposition and the analysis, without affecting the results.

$r \in \{a, na\}$ advises the principal whether to audit.

Such a mechanism is played as follows: The agent reports her type to an impartial mediator. When the reported type is θ_i, the mediator draws a contract and recommendation from the distribution π_i. The contract is publicly revealed, i.e., the agent delivers q to the principal and receives the transfer t in return. After production, the mediator privately sends the recommendation to the principal, who then decides whether to audit. In case of an audit the mediator reveals the reported type θ_i, and together with the audit finding θ this determines the penalty $P(\theta_i|\theta)$ applied to the agent.

The revelation principle establishes, that the analysis of the optimal mechanism can be confined to *incentive compatible* mechanisms, in which the agent reports her type truthfully and the principal obediently follows the mediator's recommendation.

To summarize, the timing is as follows:

1. Nature chooses the marginal cost of production, θ.
2. The agent privately learns θ.
3. The principal proposes a mechanism $\Gamma = (\pi_l, \pi_h)$.
4. The agent accepts or rejects the mechanism. In case of rejection, the game ends here.
5. The agent reports a type θ_i to the mediator.
6. The mediator draws $(t, q, P(\cdot,\cdot), r)$ from π_i makes the contract public.
7. The agent produces q and receives transfer t, and the mediator sends r to the principal.
8. The principal decides whether to audit.
9. In case of an audit, the mediator reveals θ_i. Together with the audit finding θ this determines the penalty $P(\theta_i, \theta)$.

4 Beneficial Mediation

Before analyzing optimal mechanisms in depth, I directly prove the superiority of mediation by constructing an improvement to the optimal mechanism that uses menu offers.[16] The analysis in this section yields a first insight into the underlying mechanics for beneficial mediation, that cannot be exploited by standard modes of communication.

[16]Offering a menu is equivalent to letting the agent send a single message, where the principal commits to a transfer, quantity and penalty assignment for each message; but uses the reporting strategy to update his belief about the agent's type. Bester and Strausz (2001) call this one-shot face-to-face communication, and prove that the set of messages is w.l.o.g. equal to the type space and each type reports truthfully with strictly positive probability, but this probability can be less than one.

4.1 The no-audit contract

It is instructive to first reconsider the optimal menu offer when there is no auditing by the principal. In this case, following the revelation principle, an offer is without loss of generality a pair of contracts $\{(t_l, q_l), (t_h, q_h)\}$, where in equilibrium type θ_i picks contract (t_i, q_i). More formally, the optimal menu offer is the solution to the following problem

$$\max_{t_l, q_l, t_h, q_h} \phi\big(V(q_l) - t_l\big) + (1 - \phi)\big(V(q_h) - t_h\big) \tag{2}$$

subject to

$$\begin{aligned} t_l - \theta_l q_l &\geq 0, & t_l - \theta_l q_l &\geq t_h - \theta_l q_h, \\ t_h - \theta_h q_h &\geq 0, & t_h - \theta_h q_h &\geq t_l - \theta_h q_l. \end{aligned}$$

In the solution, the participation constraint of type θ_h and the incentive constraint of type θ_l bind, which yield $t_h^{na} = \theta_h q_h^{na}$, and $t_l^{na} = \theta_l q_l^{na} + \Delta\theta q_h^{na}$. Optimal quantities are $q_l^{na} = q_l^o$ and q_h^{na} given by the first-order condition

$$V'(q_h^{na}) = \theta_h + \frac{\phi}{1 - \phi}\Delta\theta. \tag{3}$$

The no-audit contract trades off rents versus efficiency. Increasing q_h increases the efficiency by the high-cost type, but via the binding incentive constraint also increases the information rent to the low-cost type.

4.2 Menu offers under limited commitment

Auditing enables the principal to reduce the distortions imposed by the no-audit contract. Let the principal audit after the agent chose contract (t_h, q_h) with probability α. Further, a penalty P is applied to the agent whenever this audit reveals low costs. This reduces the low-cost type's payoff from reporting high costs to $t_h - \theta_l q_h - \alpha P$. Via this channel auditing reduces the information-rent to the low-cost type.

In order to ensure that this works, it is essential that the principal actually carries out the audit. To guarantee the principal's incentives ex-post, some misreporting by the agent must take place. Khalil (1997) studies menu offers that induce randomization by the agent and random audits. A main finding is the *random audit contract*, that turns out optimal for sufficiently large values of the penalty P.

The principal offers the first-best contract pair $(t_l^o, q_l^o), (t_h^o, q_h^o)$, with $t_i^o = \theta_i q_i^o$. The high-type's contract comprises a penalty assignment: the agent is penalized by P if and only if an audit

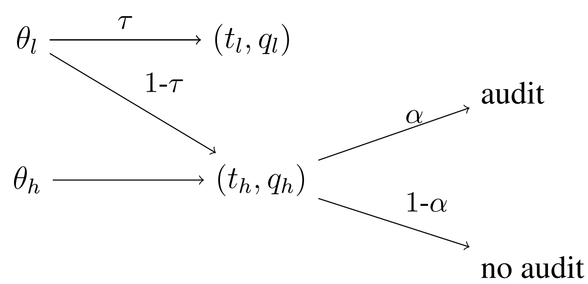

Figure 1: Optimal audit contract with random audits á la Khalil (1997).

reveals low production costs. Furthermore, the low-cost type picks (t_l^o, q_l^o) with probability τ and (t_h^o, q_h^o) otherwise. The high-cost type always picks the latter contract. An audit takes place with probability α, whenever the agent choses contract (t_h^o, q_h^o).

For an equilibrium with $0 < \alpha < 1$, the principal has to be indifferent whether to audit, i.e.,

$$\frac{\phi(1-\tau)}{\phi(1-\tau) + (1-\phi)} \cdot P = c, \tag{4}$$

which pins down the agent's strategy, $\tau = (\phi P - c)/(\phi(P - c))$.[17] The random audit contract is depicted in Figure 1, omitting the superscripts for convenience.

4.3 Menu offers are suboptimal

To illustrate why the random audit contract is not optimal when allowing for other mechanisms than menu offers, it is instructive to first convert it into a direct incentive-compatible mechanism. For this purpose, assume there is a mediator who performs all randomizations on behalf of the players. The agent only has to report her type to the mediator and the mediator recommends the principal whether to audit. More formally, the mediator uses lottery π_i, in case the agent reports θ_i. Each lottery is over the following contracts and redommendations: (1) contract (t_l^o, q_l^o), no penalties and the recommendation 'no audit', (2) contract (t_h^o, q_h^o), penalty P for low production costs and a recommendation to audit, and (3) contract (t_h^o, q_h^o), no penalties and the recommendation 'no audit'. Lotteries are

$$\pi_l = \big(\tau, (1-\tau)\alpha, (1-\tau)(1-\alpha)\big), \quad \pi_h = \big(0, \alpha, 1-\alpha\big). \tag{5}$$

This mechanism is depicted in the left panel of Figure 2. It is straightforward to verify that it is an optimal strategy for the agent to report her type truthfully and for the principal to follow the

[17]This requires $\phi P > c$, otherwise $\tau \leq 0$ - a contradiction. Lemma 1 below proves that this is indeed a necessary condition for profitable audits in the first place.

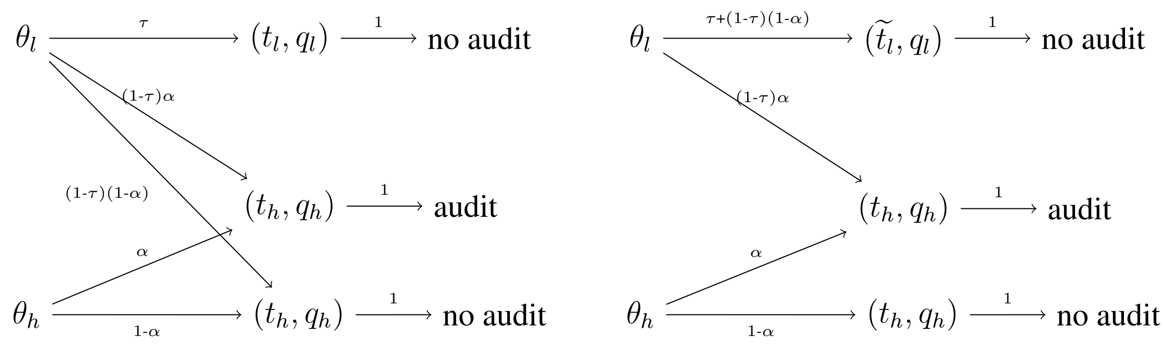

Figure 2: Random audit contract as a direct and incentive-compatible mechanism (left) and an improvement (right).

recommendation, because the random strategies form an equilibrium when the menu is offered.

The inefficiency of the random audit contract is typical for mixed equilibria: with probability $(1-\tau)(1-\alpha)$ the low-cost type lies, but walks away unpenalized. Using mediated mechanisms allows for overcoming this inefficiency. To see this, consider the alternative mechanism $\widetilde{\Gamma}$, that replaces π_l with $\widetilde{\pi}_l := \big(\tau + (1-\tau)(1-\alpha), (1-\tau)\alpha, 0\big)$. But how does this change affect players' incentives? For the principal, *provided* the agent reports truthfully, all actions remain sequentially rational. Also, reporting high costs yields the same payoff to either type of the agent, provided the principal is obedient.

Hence, the difference lies in the payoff from reporting low costs. Each type then produces q_l^o instead of q_h^o with probability $(1-\tau)(1-\alpha)$. This requires a compensation of $(1-\tau)(1-\alpha)\theta_l(q_l^o - q_h^o)$ in expected terms for the low-cost agent, in order to satisfy her incentive and participation constraint.[18] The simplest way of implementing this mark-up, is raising t_l to

$$\widetilde{t}_l = \theta_l q_l^o + \frac{(1-\tau)(1-\alpha)}{\tau + (1-\tau)(1-\alpha)} \Delta\theta q_h^o. \tag{6}$$

With the adjusted transfer, it is an optimal strategy for the low-cost type to report truthfully. Furthermore, a short calculation shows that the high-cost type does not benefit from lying.[19] Hence, there exists an equilibrium, where the agent reports truthfully and the principal follows the recommendation. This outcome is depicted in the right side of Figure 2.

Comparing the profit from the random audit contract to the new mechanism is straightforward. In the latter, the low cost type produces q_l^o instead of q_h^o with probability $(1-\tau)(1-\alpha)$ - otherwise profits from production remain unaffected. However, this comes at the cost of larger transfer payments, which amount to an expected value of $(1-\tau)(1-\alpha)\theta_l(q_l^o - q_h^o)$ to the low-cost type, as

[18] Both of which are binding in the random audit contract.

[19] The high-cost type's payoff from reporting θ_l is $(\tau + (1-\tau)(1-\alpha))(-\Delta\theta q_l^o + \frac{(1-\tau)(1-\alpha)}{\tau+(1-\tau)(1-\alpha)}\Delta\theta q_h^o) = -\tau q_l^o - (1-\tau)(1-\alpha)\Delta\theta(q_l^o - q_h^o) < 0$.

explained above. Revenues from auditing are unaffected. Comparing gains and losses, this yields a difference of

$$\phi(1-\tau)(1-\alpha)\big(V(q_l^o) - \theta_l q_l^o - V(q_h^o) + \theta_l q_h^o\big) > 0. \tag{7}$$

Consequently, whenever the optimal menu offer features random audits, there exists a mediated mechanism that yields strictly larger profit for the principal.

Mediation is beneficial, because it allows for correlating the agent's report with the recommendation to the principal. More precisely, the mechanism correlates the principal's audit decision with the agent's true type. As compared to the mixed equilibrium from the random audit contract, in the mediated mechanism the low-cost type either produces the first-best quantity or she produces q_h^o, but in the latter case she is also penalized with certainty. It requires a mediator to induce this outcome: of the three differing contracts, the low-cost type strictly prefers the lower quantity along with no audit. But the agent cannot directly pick the respective contract. Rather she can to report θ_h which, however, induces a higher likelihood of an audit, compared to reporting θ_l. The difference in audit probabilities for the respective reports deters the low-cost type from misreporting.

5 Limited commitment

The last section argued that menu offers are in general suboptimal, which again raises the question of the optimal mechanism. To address this issue, I go back to general setting with direct incentive-compatible mechanisms as introduced in section 3.

In order to set up the principal's problem, recall that each π^i has finite support, hence assume without loss of generality that the supports of π^l and π^h coincide. Denote this support

$$\Big\{\big(t_k, q_k, P_k(\cdot|\cdot)\big), r_k\Big\}_{k=1,\dots,n} \tag{8}$$

Then π_k^i is the probability that the mediator assigns contract $\gamma_k = \big(t_k, q_k, P_k(\cdot|\cdot)\big)$ and sends recommendation r_k, in case the agent reported type θ_i.

An agent of type θ, who reports θ_i to the mediator, obtains (expected) utility

$$U(\theta|\theta_i) = \sum_{k=1,\dots,n} \big[t_k - \theta q_k - \mathbb{1}_{\{r_k=a\}} P_k(\theta_i, \theta)\big]\pi_k^i. \tag{9}$$

Notice, that the agent's report affects both the probabilities π_k *and* the penalty assignments, the latter via $P_k(\theta_i, \cdot)$.[20] Furthermore, (9) is stated under the assumption that the principal obediently

[20]Therefore, the utility function does not satisfy a single crossing property and we cannot use the results obtained in Bester and Strausz (2007).

follows the mediator's recommendation. Additionally, denote $U(\theta) := U(\theta, \theta)$ the agent's utility from reporting her type truthfully.

A mechanism $\Gamma = \{\pi_l, \pi_h\}$ is *individually rational*, if for each $i \in \{l, h\}$

$$U(\theta_i) \geq 0, \tag{IR$_i$}$$

and the mechanism is *incentive compatible for the agent*, if for all $i \neq j$

$$U(\theta_i) \geq U(\theta_i|\theta_j). \tag{A-IC$_i$}$$

The mechanism is *incentive compatible for the principal*, if for all k

$$(-1)^{\mathbf{1}_{\{r_k=na\}}} \left\{ \frac{\phi \pi_k^l}{\phi \pi_k^l + (1-\phi)\pi_k^h} P_k(\theta_l, \theta_l) + \frac{(1-\phi)\pi_k^h}{\phi \pi_k^l + (1-\phi)\pi_k^h} P_k(\theta_h, \theta_h) - c \right\} \geq 0. \tag{P-IC$_k$}$$

The bracketed term is the expected revenue from auditing, if the principal observes contract γ_k and received a recommendation r_k. If $r_k = a$ this revenue has to be non-negative, whereas it has to be non-positive whenever $r_k = na$. Multiplying the revenue by '-1' if $r_k = na$ embodies this dependency.

Notice once more, that (IR$_i$) and (A-IC$_i$) assume the principal's obedience, and (P-IC$_k$) assumes the agent participates and reports her type truthfully. Lastly, a mechanism has to be feasible, i.e.,

$$\sum_{k=1,\ldots,n} \pi_k^i = 1, \quad \pi_k^i \geq 0 \; \forall k, i. \tag{FC}$$

The optimal mechanism is the solution to the following problem

$$\begin{aligned} \max_{\Gamma} \quad & \phi \sum_{k=1,\ldots,n} \left[V(q_k) - t_k + \mathbb{1}_{\{r_k=a\}} \big(P_k(\theta_l|\theta_l) - c \big) \right] \pi_k^l \\ & \qquad + (1-\phi) \sum_{k=1,\ldots,n} \left[V(q_k) - t_k + + \mathbb{1}_{\{r_k=a\}} \big(P_k(\theta_h|\theta_h) - c \big) \right] \pi_k^h \\ \text{s.t.} \quad & \text{(IR}_i\text{), (A-IC}_i\text{), (P-IC}_k\text{) and (FC).} \end{aligned} \tag{$\mathcal{P}$}$$

The main challenge in finding the optimal mechanism lies in determining the support of the probability distributions π^i, i.e., how many (different) contracts are used in equilibrium? With menu-offers there are at most as many contracts as there are types. But the last section has shown, that typically the optimal menu offer is dominated by a mechanism that uses three contracts: one per type and one for auditing purposes. Indeed, the latter is sufficiently complex for an optimal mechanism also in general.

Proposition 1. *Without loss of generality each lottery π^i is over at most two outcomes: a contract without a recommendation to audit and a contract accompanied by a recommendation to audit. When the principal is recommended to audit, he is kept just indifferent whether to follow the recommendation. An audit leads to a penalty if and only if it reveals low costs. In the optimal mechanism the high-cost type's participation constraint and the low-cost type's incentive constraint are binding, whereas the high-cost type's incentive constraint is slack.*

This result is essentially obtained as follows: First, the principal's obedience constraints when $r_k = na$ are trivially satisfied by setting $P_k(\cdot|\cdot) \equiv 0$, can thus be omitted. Next, I adopt the standard procedure of considering a relaxed problem, disregarding the high-cost type's incentive constraint.[21] Together with the first step, this allows ruling out penalties for type θ_h. Penalizing type θ_h requires a compensation via a larger transfer, but this increases the rent to type θ_l via the incentive constraint. Next, when $r_k = a$ the principal's incentive constraint must be binding: Otherwise it is profitable to reduce the share of low-cost types without violating the constraint, which in turn allows for a strict reduction of the low-cost types transfer because he pays fewer penalties in expectation.

Consequently, there are three distinct outcomes induced by the mediator in equilibrium: (1) a contract assigned exclusively to type θ_l and no recommendation to audit; (2) a contract assigned exclusively to type θ_h and no recommendation to audit; and (3) a contract assigned to both types accompanied by a recommendation to audit.[22] Using concavity of $V(\cdot)$ establishes uniqueness of each of the three variants.[23] Crucial for type (3) is the binding principal's incentive constraint, which implies that merging two differing contracts leaves the relative share of the agent's type and thereby the principal's posterior unaffected.

Proposition 1 allows to substantially reduce the problem of finding the optimal mechanism. Some notation will improve comprehensibility. Denote (t_i, q_i) the transfer-output pair of the (unique) contract that identifies type θ_i, i.e., for which $\pi_k^i > 0$ and $r_k = na$. Following the Proposition the respective penalty assignments are $P_i(\cdot|\cdot) \equiv 0$, and hence omitted in the following. Furthermore, denote (t_a, q_a) the transfer-output pair of the (unique) contract that is accompanied by a recommendation to audit, i.e., for which $r_k = a$ and $\pi_k^i > 0$. This contract entails the penalty assignment $P_a(\cdot|\cdot)$, that applies a penalty to the agent if and only if she is found to be of type θ_l - irrespective of the report.[24]

[21] I do not disregard (IR$_l$), because in the optimal contract this constraint may well be binding. So strictly speaking we do not follow the 'local'-approach, because we keep a global constraint in the problem.

[22] Proposition 1 doesn't claim the resulting contracts of type (1) and (2) differ in their quantity, but as will be shown below in Proposition 2 it is indeed optimal for the principal to propose different quantities for these cases.

[23] The argument is similar to arguing that the optimal contract under full commitment is deterministic.

[24] This perverts the traditional understanding of punishing the agent for misreporting her information. The θ_l-type is "punished" no matter what she reports. But different reports trigger a different likelihood of the punishment, which therefore still plays out as a deterrent.

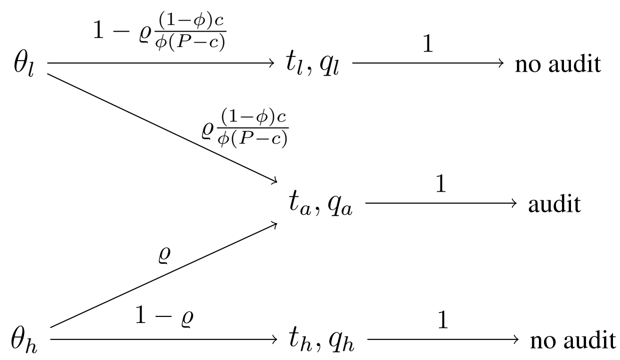

Figure 3: The optimal coordination mechanism with limited commitment.

With the above notation, a probability vector can be expressed as $\pi_i = (\pi^i_l, \pi^i_a, \pi^i_h)$. From Proposition 1, $\pi^l_h = \pi^h_l = 0$. Feasibility implies $\pi^i_i = 1 - \pi^i_a$, and the binding principal-incentive constraint yields[25]

$$\pi^l_a = \pi^h_a \frac{(1-\phi)c}{\phi(P-c)}. \tag{10}$$

Hence, of the four non-zero probabilities only one is a free variable, that exactly determines the other three. Denote

$$\varrho := \pi^h_a. \tag{11}$$

Figure 3 illustrates the mechanism, using the notation introduced above.

The variable ϱ represents the audit probability in the mechanism. Obviously, whenever $\varrho = 0$ no audits are conducted. For $\varrho = 1$ a high-cost report certainly triggers an audit, consequently this case is referred to as ”certain audits“. When $\varrho \in (0, 1)$ audits take place in equilibrium, but conditional on either report this probability is less than one - this case is referred to as ”random audits“.

When the mediator recommends audits, it is essential that the principal does not observe the agent's report. The latter is only revealed after an audit, when the report together with the audit finding determines the penalty applied to the agent. Due to limited commitment, the mechanism has to recommend an audit for some low- and some high-cost types to guarantee the principal's obedience. If the share of high-cost types is too high, the principal will not follow the recommendation. However, if the share of low-cost types is too high it is too costly to incentivize the agent to report truthfully. Following Proposition 1 it is optimal to keep the principal just indifferent whether to follow the recommendation. The resulting indifference condition and the feasibility requirement determines all probabilities but one.

[25]At this point we might still have $\pi^l_a > 1$. The proof of Lemma 1 below, implies that $\phi P > c$ whenever $\pi^h_a > 0$. This implies that all probabilities are indeed between zero and one.

Using the above notation, the principal's profit from offering the mechanism is

$$\begin{aligned}&\phi\left(1-\varrho\frac{(1-\phi)c}{\phi(P-c)}\right)\big(V(q_l)-t_l\big)+\left(\phi\varrho\frac{(1-\phi)c}{\phi(P-c)}+(1-\phi)\varrho\right)\big(V(q_a)-t_a\big)\\&\quad+(1-\phi)(1-\varrho)\big(V(q_h)-t_h\big).\end{aligned}\tag{12}$$

Notice that revenues from auditing disappeared. Because the obedience constraint is binding, the principal is indifferent between auditing and not whenever recommended to do so, and, hence, in equilibrium audit revenues equal zero. In line with Proposition 1, the optimal mechanism maximizes (12) with respect to the contract variables $t_l, q_l; t_a, q_a; t_h, q_h$ and ϱ, under the following constraints: the (binding) participation constraint of type θ_h

$$\varrho(t_a-\theta_h q_a)+(1-\varrho)(t_h-\theta_h q_h)=0,\tag{13}$$

the (binding) incentive constraint of type θ_l

$$\begin{aligned}\left(1-\varrho\frac{(1-\phi)c}{\phi(P-c)}\right)(t_l-\theta_l q_l)+\varrho\frac{(1-\phi)c}{\phi(P-c)}(t_a-\theta_l q_a-P)=\\ \varrho(t_a-\theta_l q_a-P)+(1-\varrho)(t_h-\theta_l q_h).\end{aligned}\tag{14}$$

and the participation constraint of type θ_l, which may or may not be binding,

$$\left(1-\varrho\frac{(1-\phi)c}{\phi(P-c)}\right)(t_l-\theta_l q_l)+\varrho\frac{(1-\phi)c}{\phi(P-c)}(t_a-\theta_l q_a-P)\geq 0.\tag{15}$$

Substituting t_l, t_a and t_h from the former two binding constraints, the principal's profit can be written as a function $\mathcal{V}(\varrho, q_l, q_a, q_h)$:[26]

$$\begin{aligned}\mathcal{V}(\varrho,q_l,q_a,q_h)=\;&\underbrace{\phi\big(V(q_l)-\theta_l q_l-\Delta\theta q_h\big)+(1-\phi)\big(V(q_h)-\theta_h q_h\big)}_{\text{'no-audit`-profit}}\\&+\varrho\Bigg[-\underbrace{\frac{(1-\phi)c}{P-c}\big(V(q_l)-\theta_l q_l-V(q_a)+\theta_l q_a\big)}_{l\text{-type } q_l\to q_a}-\underbrace{\phi\Delta\theta(q_a-q_h)}_{\text{rent for } l\text{-type}}\\&+\underbrace{(1-\phi)\big(V(q_a)-\theta_h q_a-V(q_h)+\theta_h q_h\big)}_{h\text{-type } q_h\to q_a}+\underbrace{\frac{\phi P-c}{P-c}P}_{\text{deterring } P}\Bigg]\end{aligned}\tag{16}$$

[26] The two binding constraints determine three variables, because only type-specific transfers matter. In fact, there is a continuum of transfer-pairs t_h, t_a that satisfy (13). For any such pair there exists a unique t_l such that (14) is satisfied. The expected transfer is the same for all these transfer-triples.

The principal now maximizes (16) subject to (15). From (16) the impact of audits becomes clear. When $\varrho = 0$, the principal never audits and the expression coincides with the virtual surplus formulation in the case without auditing. If auditing takes place in equilibrium, the mechanism has to create the right incentives for the principal. To this end the mechanism introduces a new contract $(t_a, q_a, P_a(\cdot|\cdot))$ along with the recommendation to audit. As explained earlier a particular ratio of low- and high-cost types is required to meet the principal's incentive constraint.

Introducing this new contract has several effects on the principal's profit: Each type of the agent produces q_a instead of q_i. Whether this creates a loss or a gain, depends on whether q_i or q_a is more efficient for the respective type. Notice that the respective weights differ, because different shares of low- and high-cost types are assigned q_a. Second, letting the high-cost type produce q_a instead of q_h directly affects the rent for type θ_l via the (binding) incentive constraint - this is captured in the third expression of (16). Finally, the audit itself plays out as a deterrent. Because only low-cost types are penalized, this allows for a rent-reduction. The fraction $(\phi P - c)/(P - c)$ represents the difference in likelihood for q_a when reporting low, resp., high costs. The threat of an audit deters the low-cost type, whenever audits are more likely after a misreport - this is exactly the case when $\phi P > c$, which I will discuss in more detail below.

Intuitively, audits are profitable whenever the term in square brackets in expression (16) is positive. Though this neglects a possibly binding (15), it nevertheless contains a considerable element of truth, as the following lemma shows.[27]

Lemma 1. *There exists a unique value $P^* < \infty$, satisfying*

$$\max_{q_l, q_a, q_h} \left\{ \frac{(1-\phi)c}{P^* - c}\big(V(q_l) - \theta_l q_l - V(q_a) + \theta_l q_a\big) + \phi \Delta\theta (q_a - q_h) \right. \\ \left. -(1-\phi)\big(V(q_a) - \theta_h q_a - V(q_h) + \theta_h q_h\big) - \frac{\phi P^* - c}{P^* - c} P^* \right\} = 0 \tag{17}$$

such that the optimal mechanism entails $\varrho^* > 0$ if and only if $P > P^*$. In particular, we have $\phi P^* > c$.

Notice from Lemma 1, a necessary condition for profitable audits is $\phi P^* > c$. Why is this the case? Audits are supposed to relax the low-cost types incentive constraint. In equilibrium, if at all, audits occur after either type report. Hence, the need for deterrence requires strictly more audits after a high-cost report. On the other hand, auditing requires incentives for the principal. Denote ϕ_a the principal's posterior when receiving a recommendation to audit. Following Proposition 1

[27] This result goes beyond Lemma 2 in Khalil (1997), because it defines an exact condition for profitable audits. Khalil argues that "for high enough penalties [..] the audit contract will dominate the Baron-Myerson contract.". For his Lemma 2 he uses a limiting result, showing that as $P \to \infty$ the profit from the audit contract approximates the first-best.

the principal is kept just indifferent, hence $\phi_a P = c$. Thus, auditing is more frequent after a high-cost report if and only if $\phi P > c$. Incidentally, this condition also guarantees that all probabilities π_k^i of the optimal contract lie between zero and one, in particular $\pi_a^l \in [0, 1)$ for all $\varrho \in [0, 1]$, see (10).

Lemma 1 further allows for a comparison with the full commitment benchmark. Following Baron and Besanko (1984), the principal audits under full commitment whenever $\phi P > (1-\phi)c$.[28] Because $\phi P^* > c$ we trivially have $\phi P^* > (1 - \phi)c$ and audits are thus already profitable under full commitment. Consequently, with limited commitment the principal uses his audit technology only for larger levels of the deterrent than under full commitment. In particular, with the latter an audit may well be unprofitable ex-post even if the principal knew the agent's type. To see this, observe that the condition $\phi P > (1 - \phi)c$ does not require $P > c$.

Whenever $P < P^*$, Lemma 1 implies that the optimal mechanism corresponds to the no-audit contract from section 4.1. If at the same time $\phi P^* > (1 - \phi)c$ the commitment problem has a drastic consequence: Though audits are profitable and carried out under full commitment, they cannot be profitably used when commitment is limited.

For the remainder assume the deterrent is large enough, i.e. $P > P^*$, such that audits are used in the optimal mechanism under limited commitment. Define the following three mechanisms:

(RE) $q_l^{RE} = q_l^o$, $\varrho^{RE} = 1$, and $q_a^{RE} > q_h^{na}$ given implicitly by

$$V'(q_a^{RE}) = \theta_h + \frac{\phi P - c}{(1-\phi)P}\Delta\theta, \tag{18}$$

(OA) $q_l^{OA} = q_l^o$, $\varrho^{OA} = 1$, and $q_a^{OA} = P/\Delta\theta$,

(RA) $q_l^{RA} = q_l^o$, $\varrho^{RA} = \big(\Delta\theta q_h^{RA}\big)/\big(P - \Delta\theta(q_a^{RA} - q_h^{RA})\big) \in (0, 1)$. Quantities are $q_h^{na} < q_h^{RA} < q_a^{RA} < q_h^o$, and

$$V'(q_h^{RA}) - \theta_h = \frac{P}{P-c}\big(V'(q_a^{RA}) - \theta_h\big) + \frac{c}{P-c}\Delta\theta. \tag{19}$$

The following Proposition states existence and uniqueness of an optimal mechanism, and shows that it is either of the three above mentioned types.

Proposition 2. *An optimal audit mechanism exists and it is unique. There exists a unique threshold $\underline{P}^m$, such that the optimal audit mechanism is of type (RE) whenever $P \leq \underline{P}^m$, otherwise it is either (OA) or (RA). For $P \geq \Delta\theta q_h^o$ the optimal audit mechanism is random, i.e., of type (RA). The agent obtains a strictly positive rent if and only if $P < \underline{P}^m$, otherwise* (15) *is binding.*

[28] The intuition is: Via the penalty P, an audit reduces the rent to the low-cost type by at most P. This increases the (expected) profit by ϕP, but comes at a cost of $(1 - \phi)c$. See also Kofman and Lawarrée (1993) for a derivation of the cutoff in a similar model.

The first type of audit mechanism is referred to as (RE), abbreviating 'rent extraction'. The principal uses audits to reduce the rent left to the low-cost type. As long as (15) is not binding, the proof argues that optimal quantities are independent of ϱ. But then the principal's objective is linear in ϱ and hence $\varrho^{RE} = 1$. The threshold value $\underline{P}^m$ is determined by the unique P for which (15) is binding, when q_a is determined by (18). It is worth noting, that although $q_a^{RE} > q_h^{na}$, it is not the case that the principal uses audits to increase the efficiency of the allocation. Rather do both types of the agent produce q_a^{RE} in equilibrium, and the distortion takes this into account.

As soon as P is sufficiently large, the agent's rent is reduced to zero in the optimal mechanism. Now distortions on quantities are subsequently reduced. In mechanism (OA) - output adjustment - this is done exclusively via increasing q_a. In particular, this type of mechanism keeps $\varrho = 1$ and consequently only two differing quantities are produced in equilibrium.

In the third type of audit mechanism ('random audit' or (RA)) the principal finds it optimal to reduce ϱ - the ex-ante probability of an audit for a high-cost type. Reducing ϱ has the benefit for the principal that the low-cost type produces more often the efficient quantity q_l^o. On the other hand, it requires a reduction of q_a and/or q_h in order to keep (15) valid. Because both of these quantities are below q_h^o, this is costly for the principal. Hence, the principal trades off the benefits of inducing fewer audits against the cost of imposing distortions on the efficient type's production.

To better understand the underlying trade-off, it is useful to solve (15) for ϱ

$$\varrho = \frac{\Delta\theta q_h}{P - \Delta\theta(q_a - q_h)}, \tag{20}$$

and rewrite the principal's objective (16) as follows

$$\begin{aligned}\mathcal{V}(\varrho, q_l, q_a, q_h) = (1-\varrho)\left\{\phi\big(V(q_l) - \theta_l q_l - \Delta\theta q_h\big) + (1-\phi)\big(V(q_h) - \theta_h q_h\big)\right\} \\ +\varrho\left\{\frac{\phi P - c}{P - c}\big(V(q_l) - \theta_l q_l - \Delta\theta q_a + P\big) + \frac{(1-\phi)P}{P-c}\big(V(q_a) - \theta_h q_a\big)\right\}.\end{aligned} \tag{21}$$

The latter is a weighted average of the principal's virtual surplus from offering a contract without audits and the virtual surplus of offering a contract with sure audits under limited commitment, i.e., where the high-cost type is always audited. The weight is ϱ, which corresponds to the probability of assigning contract (t_a, q_a). Increasing q_a, resp. q_h, therefore has the usual *rent effect*. But as discussed earlier, for sufficiently large penalties the mechanism leaves no rent to the agent, because the principal now uses more audits to reduce the rent back to zero. This is implied by (20), which shows that ϱ increases both with q_a, and q_h. But using audits more frequently requires the mechanism puts more weight on the contract (t_a, q_a). This comes at the cost of having each agent's type produce q_a instead of q_i - the *audit-cost effect*.

What becomes crucial in comparing the rent- and the audit-cost-effect is, that the latter has the same (relative) magnitude for changes in q_a, resp. q_h.[29] But increasing q_h increases the rent by $\phi\Delta\theta$, whereas increasing q_a increases the rent only by $(\phi P - c)/(P - c)\Delta\theta < \phi\Delta\theta$. Thus, the rent-effect is stronger for q_h and hence $q_a > q_h$.

With increasing P, the audit-cost-effect is weakened, because only a slight change in audit probability already has a severe impact on the agent's rent via the large penalty.[30] Therefore, quantities q_a and q_h increase with P and ϱ decreases. Also the difference between the rent-effects decreases with P, so the difference in the two quantities decreases with P. This can also be seen from (19), because $c/(P - c)\Delta\theta$ vanishes for large P.

As P approaches infinity, both q_a and q_h approximate q_h^o. However, for finite P it always holds that $q_h < q_a < q_h^o$. If $q_h = q_a = q_h^o$, then a reduction of these quantities has no first-order effect on the profit from interacting with the high-cost type. However, it allows for a strict reduction of ϱ, which is first-order beneficial due to the gain from shifting production of the low-cost type from q_a to $q_l = q_l^o$.

Proposition 2 also determines the type of the audit mechanism for $P \leq \underline{P}^m$ as well as $P \geq \Delta\theta q_h^o$. A general inference for $P \in (\underline{P}^m, \Delta\theta q_h^o)$ depends strongly on the shape of $V(\cdot)$. In general ϱ^* is not monotone on this range. The example below provides a case where ϱ^* is nevertheless monotone, hence there exists $\overline{P}^m \in (\underline{P}^m, \Delta\theta q_h^o)$ such that $\varrho^* < 1$ if and only if $P > \overline{P}^m$.

Example 1. Let $V(q) = 2\sqrt{q}$ and the cost parameters be given by $\theta_l = 1$, resp., $\theta_h = 2$. Then $q_l^o = 1$ and $q_h^o = 1/4$. furthermore, assume $\phi = 1/2$, which yields $q_h^{na} = 1/9$ and the welfare from the no-audit contract equals $\mathcal{V}^{na} = 2/3$.
Now consider audits and assume $c = 0.01$. The threshold-value P^* for profitable audits can be computed as $P^* \approx 0.0758$. Furthermore, we have $\underline{P}^m \approx 0.1241$. Consequently we have $\varrho^* = 1$ for all $P \in (P^*, \underline{P}^m]$. Lengthy calculations show that there exists a unique value $\overline{P}^m \approx 0.2387$ such that $\varrho^* = 1$ also for $P \in (\underline{P}^m, \overline{P}^m]$. On this interval, (15) binds. Only for $P > \overline{P}^m$, we have $\varrho^* < 1$. Notice further that $\overline{P}^m < \Delta\theta q_h^o$.

6 Implementation of the Optimal Contract

This section briefly discusses ways of implementing the optimal mechanism. A natural question to ask is whether and when the optimal mechanism can be implemented using menu offers.

When $\varrho^* = 0$ the trivial answer is yes - the optimal mechanism is just the optimal menu without auditing. Slightly more subtle is the case where the optimal mechanism prescribes $\varrho^* = 1$. Because

[29]Relative in the following sense: Differentiating ϱ yields $\partial\varrho/\partial q_h = (1 - \varrho)\varrho/q_h$ and $\partial\varrho/\partial q_a = \varrho^2/q_h$. When differentiating (21) with respect to quantities, the rent-effect has weight $1 - \varrho$ for q_h, resp. ϱ for q_a. Factoring these weights out, the audit-cost-effect is the same, irrespective of the quantity.

[30]The relationship is in general not monotone. It depends on the shape of $V(\cdot)$, as already for the full commitment case. For large P this can be shown to hold in general, irrespective of the shape of $V(\cdot)$.

in this case either type report triggers a randomization over the same two contracts, the low-cost type's binding incentive constraint implies she is not only indifferent between the two lotteries, but also between the contracts itself. Hence, provided the principal always audits when the agent picks (t_a, q_a), there exists a PBE where the low-cost type randomly picks a contract. Furthermore, if this randomization is exactly as in the audit mechanism, the principal is indifferent whether to audit. This shows that there exists a PBE with a menu offer which yields the principal the same expected profit as the optimal mechanism with a mediator.

Whenever $\varrho^* < 1$, however, menu offers fail to implement the optimal mechanism. The reason is, that there exist no transfers t_l, t_a and t_h such that the low-cost type is indifferent between contracts (t_l, q_l) and (t_a, q_a), whereas the high-cost type is indifferent between (t_h, q_h) and (t_a, q_a).[31] Also, the randomization cannot be executed by the principal in form of a stochastic mechanism, because the implied knowledge of the agent's report when deciding upon an audit. But the optimal communication mechanism requires that the principal does not know the agent's report after the randomization realized in production of q_a.

The following corollary formalizes the last statements.

Corollary 1. *The optimal communication mechanism can be implemented via offering a (potentially stochastic) menu if and only if* $\varrho^* \in \{0, 1\}$.

Hence, implementation for $\varrho^* \in \{0, 1\}$ is straightforward and does in particular not require a mediator. When $\varrho^* \in (0, 1)$, some form if indirect communication between agent and principal is required. Notice, however, that the confidentiality of the recommendation to the principal is not necessary. This is due to the fact that only one party has an action which is recommended. In general, confidentiality is indispensable when the mechanism aims at correlating simultaneous actions of several players, which is not the case in the simple principal-agent framework studied in this article. Similar to results from the literature on cheap-talk, the optimal mechanism with $\varrho^* \in (0, 1)$ can be implemented using noisy communication, where an agent's (truthful) type report is translated into a noisy message. The principal uses the received message to update his belief about the agent's type and decides whether to audit. The optimal mechanism can then be implemented using a three-message noisy communication device, where the principal commits to contract (t_l, q_l) for message m_1, contract (t_a, q_a) for message m_2, and contract (t_h, q_h) for message m_3. Bester and Strausz (2007) study noisy communication devices of this kind, but they leave open the question of whether there exist mechanisms outside their framework (e.g., that use a mediator) that achieve higher profits.

[31] In fact, if implementation via a menu was possible, following Bester and Strausz (2001) there existed an equivalent menu with only two contracts. But this contradicts optimality of the audit mechanism, because $q_l > q_a > q_h$.

7 Conclusion

This paper derives the optimal mediated mechanism in a principal-agent framework when the principal cannot commit to an audit strategy. As compared to the previous literature, I do not restrict the analysis to menu offers, but instead allow for general mechanisms and make use of a revelation principle. The latter states that I can restrict to direct and incentive-compatible mechanisms that use a mediator. The agent reports her private information to the mediator, who assigns contracts and recommends the principal whether to audit.

As an important result I show that the principal's problem of finding the optimal mechanism can be further simplified: for each type the mediator randomizes over at most two contracts - a type-dependent base contract and the audit contract, which is accompanied by a recommendation to audit. This simplification for the first time allows transforming the principal's problem into a tractable optimization problem and for characterizing the optimal mechanism. I prove existence of a threshold value, such that the optimal mechanism uses audits if and only if the penalty exceeds this threshold. Furthermore, for sufficiently large penalties the optimal mechanism uses three different contracts, despite there being only two types of the agent. Such a mechanism cannot be implemented with menu offers, but requires indirect communication via an impartial mediator or a noisy channel.

The optimal mechanism reveals new insights in the beneficial role of mediation in contracting with limited commitment. Using a mediator allows for correlating the agent's report with the principal's action. Though this feature is known from the study of communication equilibria, it has not been applied to contract theory yet. The results of this paper therefore provide new insights into both the structure of optimal mechanisms and the analysis itself. Building on these insights may help studying mediation in different models, such as those of dynamic contracting with limited commitment (e.g., Laffont and Tirole (1988)), bilateral trade (e.g., Skreta (2006)) or auction design without commitment (e.g., Vartiainen (2013) and Skreta (2013)).

I study a stylized two-type model. A natural question that arises is whether the results change when allowing for more types. The beneficial role of mediation carries over, but the analysis gets easily intractable. Already the full commitment case is messy to analyze with more than two types, see for instance Baron and Besanko (1984). The problem lies in identifying the binding constraints, because the possibility of an audit effectively transforms the one-dimensional screening problem into a multi-dimensional screening problem.

Furthermore, I make two important assumptions on the audit technology: audits perfectly reveal the agent's type and the principal can only impose a punishment of P or no punishment at all. The latter assumption is known as the *maximum-punishment principle* for the case of full commitment. In a companion paper, Pollrich (2015), I show that relaxing either of the two assumptions is not innocuous. Nevertheless, my main contribution – that mediation is beneficial and menu offers are sub-optimal – prevails also when considering imperfect audit technologies or when allowing for intermediate penalties.

Another interesting direction for future research lies in multistage communication. For instance can the principal offer several menus in subsequent rounds, where the agent has the option to pick one contract or move to the next round. Between rounds principal and agent play a jointly controlled lottery. In cheap talk a similar structure is already useful and can even achieve the outcome from using a mediator, but is an open question to what extend this carries over to a contracting problem with transfers.

Lastly, I study a model of ex-ante private information, i.e., the agent knows her type *before* she is offered a mechanism. There is also a large literature on the related problem with *interim* private information, where the agent learns her type only *after* signing the contract. Named examples include: insurances contract with audits, debt contracts, or more general the literature on costly state verification. The main driver of beneficial mediation – correlating the agent's report with the recommendation to the principal – is equally applicable. However, it must be demonstrated that there is a strict gain from using a mediator rather than simple mechanisms where the agent reports her information directly to the principal.

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A Proofs of Section 5

Proof of Proposition 1. We first set up a relaxed problem and later verify that its solution also solves ($\mathcal{P}$).

First, observe that when $r_k = na$, obedience can be easily guaranteed by setting $P_k(\cdot|\cdot) \equiv 0$. It is therefore without loss of generality to focus on obedience constraints when $r_k = a$, i.e.,

$$\left\{ \frac{\phi\pi_k^l}{\phi\pi_k^l + (1-\phi)\pi_k^h} P_k(\theta_l, \theta_l) + \frac{(1-\phi)\pi_k^h}{\phi\pi_k^l + (1-\phi)\pi_k^h} P_k(\theta_h, \theta_h) - c \right\} \geq 0 \qquad \text{(OC}'\text{)}$$

for all k such that $r_k = a$.

Next, we disregard (IC$_h$), as it is standard in solving incentive problems.[32] Finally, a simplification that helps us to substantially reduce the complexity of the following analysis is to focus on *type-dependent* transfers. In particular, we substitute $\sum_k t_k\, \pi_k^i$ by T_i. This allows for more flexibility in the principal's problem and turns out to be analytically more tractable. Notice however that this constitutes a purely theoretical simplification, because the principal will observe the transfer paid to the agent. Hence, at the end of our analysis we shall point out how to re-transform

[32] The problem here is that the screening problem is essentially multi-dimensional through the impact of the penalty schemes. Potential penalties for the inefficient type may render (IC$_h$) binding. By assuming the constraint to be slack, we can easily rule out these penalties, which ultimately helps justifying the assumption in the first place.

the type-dependent transfers into allocation-specific transfers. The agent's individual rationality constraints now read as

$$T_i - \sum_{k=1,\ldots,n} \left[\theta_i q_k + \mathbb{1}_{\{r_k=a\}} P(\theta_i|\theta_i)\right]\pi_k^i \ \geq 0, \tag{IR$'_i$}$$

for $i = l, h$, and the efficient type's incentive compatibility constraint is

$$T_l - \sum_{k=1,\ldots,n} \left[\theta_l q_k + \mathbb{1}_{\{r_k=a\}} P(\theta_l|\theta_l)\right]\pi_k^l \ \geq \ T_h - \sum_{k=1,\ldots,n} \left[\theta_l q_k + \mathbb{1}_{\{r_k=a\}} P(\theta_h|\theta_l)\right]\pi_k^h. \tag{P-IC$'_l$}$$

The principal's profit from offering contract Γ can now be stated as

$$\begin{aligned}\mathcal{V}(\Gamma) = \quad & \phi \sum_{k=1,\ldots,n} \left[V(q_k) + \mathbb{1}_{\{r_k=a\}}(P_k(\theta_l|\theta_l) - c)\right]\pi_k^l - \phi T_l \\ & + (1-\phi) \sum_{k=1,\ldots,n} \left[V(q_k) + \mathbb{1}_{\{r_k=a\}}(P_k(\theta_h|\theta_h) - c)\right]\pi_k^h - (1-\phi)T_h.\end{aligned} \tag{22}$$

The auxiliary problem we solve in the following is

$$\max_{\Gamma} \mathcal{V} \quad \text{s.t. (IR}'_i\text{), (P-IC}'_l\text{), (OC}'\text{), (FC)} \quad \text{for } i = l, h \text{ and all } k \tag{$\mathcal{P}'$}$$

The proof is now a sequence of intermediate results. First observe that $P_k(\theta_l|\theta_h)$ appears in none of the constraints of problem ($\mathcal{P}'$), and hence we can set $P_k(\theta_l|\theta_h) = 0$ without loss of generality. The following two Lemmas rule out penalties for the inefficient type.

Lemma A.1. *If Γ^* is a solution to problem ($\mathcal{P}'$) and there exists k such that $r_k = a$ and $P_k(\theta_h|\theta_h) = P$, then $P_k(\theta_l|\theta_l) = 0$.*

Proof. Assume the contrary, i.e., $P_k(\theta_l|\theta_l) = P$. Consider the alternative contract $\widetilde{\Gamma}$, where each T_i is replaced by $\widetilde{T}_i = T_i - \pi_k^i P$. Furthermore, set $\widetilde{P}_k(\theta|\theta') = 0$ for all θ, θ', and let $\widetilde{r}_k = na$. In all other respects $\widetilde{\Gamma}$ coincides with Γ^*. It is straightforward to verify that $\widetilde{\Gamma}$ satisfies all constraints of problem ($\mathcal{P}'$) and yields expected profit $\mathcal{V}(\widetilde{\Gamma}) = \mathcal{V}(\Gamma^*) + (\phi\pi_k^l + (1-\phi)\pi_k^h)c > \mathcal{V}(\Gamma^*)$, hence Γ^* was not optimal. □

Lemma A.2. *If Γ^* is a solution to problem ($\mathcal{P}'$), then without loss of generality $P_k(\theta_h|\theta_h) = 0$ for all k.*

Proof. If k is such that $r_k = na$, then there is indeed no loss in setting $P_k(\theta_h|\theta_h) = 0$, as argued above. Now, assume $r_k = a$ and $P_k(\theta_h|\theta_h) = P$. By Lemma A.1 we have $P_k(\theta_l|\theta_l) = 0$. Consider the alternative contract $\widetilde{\Gamma}$ with $\widetilde{P}_k(\cdot|\cdot) \equiv 0$ and $\widetilde{r}_k = na$. Furthermore, let $\widetilde{T}_h = T_h - \pi_k^h P$.

It is again straightforward to verify all constraints of problem ($\mathcal{P}'$) for contract $\widetilde{\Gamma}$. Furthermore, $\mathcal{V}(\widetilde{\Gamma}) = \mathcal{V}(\Gamma^*) + (1-\phi)\pi_k^h P + \phi\pi_k^l c - (1-\phi)\pi_k^h(P-c) = \mathcal{V}(\Gamma^*) + \big(\phi\pi_k^l + (1-\phi)\pi_k^h\big)c > \mathcal{V}(\Gamma^*)$, hence Γ^* was not optimal. □

The previous two Lemmas imply that we can set $P_k(\cdot|\theta_h) = 0$ for all k. Furthermore, whenever $r_k = a$ we must have $P_k(\theta_l|\theta_l) = P$ in order to guarantee obedience, and we can set $P_k(\theta_h|\theta_l) = P$ if $r_k = a$ without loss of generality, because it only strengthens (P-IC$_l'$).

The following Lemma argues that the principal is kept indifferent whether to obey the mediators recommendation to audit.

Lemma A.3. *If $r_k = a$, then the respective obedience constraint is binding.*

Proof. Fix k such that $r_k = a$. As argued above we have $P_k(\cdot|\theta_l) = P$ and zero otherwise. Assume by contradiction, that the principal strictly prefers to obey the mediator's recommendation, i.e.,

$$\frac{\phi\pi_k^l}{\phi\pi_k^l + (1-\phi)\pi_k^h} P > c.$$

Consider the alternative contract $\widetilde{\Gamma}$ with $\widetilde{\pi}_k^l = \pi_k^l - \varepsilon$ and set $\widetilde{\pi}_{n+1}^l = \varepsilon$ and $\widetilde{\pi}_{n+1}^h = 0$. Further, let $\widetilde{q}_{n+1} = q_k$, $\widetilde{r}_{n+1} = na$ and $\widetilde{P}_{n+1}(\cdot,\cdot) \equiv 0$. In all other respect the contracts $\widetilde{\Gamma}$ and Γ coincide. Provided ε is small, the above obedience constraint is still valid. Setting $\widetilde{T}_l = T_l - \varepsilon P$ keeps the low-cost types payoff from reporting truthfully unaffected. Then also (P-IC$_l'$) holds. All remaining constraints are unaffected. Finally, we have $\mathcal{V}(\widetilde{\Gamma}) = \mathcal{V}(\Gamma) + \phi\varepsilon P - \phi\varepsilon(P-c) = \mathcal{V}(\Gamma) + \phi\varepsilon c$. Consequently, Γ was not optimal. □

The next three lemmas are concerned with the support of each π^i.

Lemma A.4. *Suppose Γ^* is a solution to problem ($\mathcal{P}'$) and there exist $k \neq k'$ such that $\pi_k^l > 0$ and $\pi_{k'}^l > 0$ as well as $r_k = r_{k'} = na$. Then $q_k = q_{k'}$.*

Proof. Assume the contrary, i.e., $q_k \neq q_{k'}$. Consider the alternative mechanism $\widetilde{\Gamma}$ with $\widetilde{q}_{n+1} = (\pi_k^l q_k + \pi_{k'}^l q_{k'})/(\pi_k^l + \pi_{k'}^l)$ and $\widetilde{\pi}_{n+1}^l = \pi_k^l + \pi_{k'}^l$, as well as $\widetilde{\pi}_{k'}^l = \widetilde{\pi}_k^l = \widetilde{\pi}_{n+1}^h = 0$. Furthermore, let $\widetilde{r}_{n+1} = na$ and $\widetilde{P}_{n+1}(\cdot|\cdot) \equiv 0$. In all other respects the contracts Γ^* and $\widetilde{\Gamma}$ coincide. Because the agent's constraints are linear in quantities, they are satisfied for contract $\widetilde{\Gamma}$, as they were satisfied for contract Γ^*. The difference in profits from the two contracts is $\mathcal{V}(\widetilde{\Gamma}) - \mathcal{V}(\Gamma^*) = (\pi_k^l + \pi_{k'}^l)V(\widetilde{q}_k) - \pi_k^l V(q_k) - \pi_{k'}^l V(q_{k'})$ which is strictly positive, because V is strictly concave. and $\widetilde{q}_k$ is a convex combination of q_k and $q_{k'}$. □

Lemma A.5. *Suppose Γ^* is a solution to problem ($\mathcal{P}'$) and there exist $k \neq k'$ such that $\pi_k^h > 0$ and $\pi_{k'}^h > 0$ as well as $r_k = r_{k'} = na$. Then $q_k = q_{k'}$.*

Proof. The proof repeats the steps from Lemma A.4. □

Lemma A.6. *Suppose* Γ^* *is a solution to problem* $(\mathcal{P}')$ *and there exist* $k \neq k'$ *such that* $\pi_k^l > 0$ *and* $\pi_{k'}^l > 0$ *as well as* $r_k = r_{k'} = a$. *Then* $q_k = q_{k'}$.

Proof. Suppose the contrary, i.e., $q_k \neq q_{k'}$. Then also $\pi_k^h > 0$ and $\pi_{k'}^h > 0$ by Lemma A.3. Consider the alternative contract $\widetilde{\Gamma}$ with

$$\widetilde{q}_k := (q_k \pi_k^l + q_{k'} \pi_{k'}^l)/(\pi_k^l + \pi_{k'}^l),$$

and $\widetilde{\pi}_k^l = \pi_k^l + \pi_{k'}^l$, as well as $\widetilde{\pi}_k^h = \pi_k^h + \pi_{k'}^h$. Further let $\widetilde{\pi}_{k'}^l = \widetilde{\pi}_{k'}^h = 0$, and $\widetilde{r}_{k'} = na$. In all other respects $\widetilde{\Gamma}$ coincides with Γ^*. From the binding obedience constraints we get

$$\pi_k^h = \frac{\phi(P-c)}{(1-\phi)c}\pi_k^l, \qquad \pi_{k'}^h = \frac{\phi(P-c)}{(1-\phi)c}\pi_{k'}^l,$$

and therefore

$$\widetilde{\pi}_k^h \widetilde{q}_k = (\pi_k^h + \pi_{k'}^h)\frac{q_k \pi_k^l + q_{k'}\pi_{k'}^l}{\pi_k^l + \pi_{k'}^l} = \frac{\phi(P-c)}{(1-\phi)c}(q_k \pi_k^l + q_{k'}\pi_{k'}^l) = \pi_k^h q_k + \pi_{k'}^h q_{k'}.$$

Thus, as in the proof of Lemma A.4, validity of the agent's constraints for contract Γ^* implies validity of the same constraints for contract $\widetilde{\Gamma}$. Furthermore, the obedience constraint for $\widetilde{r}_k = a$ is satisfied. To see this, notice that we have $\phi\pi_k^l P = (\phi\pi_k^l + (1-\phi)\pi_k^h)c$, as well as $\phi\pi_{k'}^l P = (\phi\pi_{k'}^l + (1-\phi)\pi_{k'}^h)c$, from the obedience constraints of contract Γ^*. Adding the two equalities yields $\phi\widetilde{\pi}_k^l P = (\phi\widetilde{\pi}_k^l + (1-\phi)\widetilde{\pi}_k^h)c$, which is equivalent to the respective obedience constraint for contract $\widetilde{\Gamma}$. Trivially, $\widetilde{r}_{k'} = na$ is followed. Hence, contract $\widetilde{\Gamma}$ satisfies all constraints of problem $(\mathcal{P}')$. By the same arguments as used in Lemma A.4, contract $\widetilde{\Gamma}$ yields strictly larger profits then contract Γ^*. □

We thus have shown, that the support of each π^i has at most two elements. This gives us in total at most three distinct outcomes - $(q_l, P_l(\cdot|\cdot), r_l), (q_h, P_h(\cdot|\cdot), r_h)$ and $(q_a, P_a(\cdot|\cdot), r_a)$. Furthermore, we have $\pi_h^l = \pi_l^h = 0$, i.e., the first outcome is never assigned to a report θ_h and the second outcome is never assigned to a report θ_l. Additionally,we have $r_l = r_h = na$ and can consequently set $P_i(\cdot|\cdot) \equiv 0$ for $i = l, h$. Also, $P_a(\cdot|\theta_l) \equiv P$ and $P_a(\cdot|\theta_h) \equiv 0$ and $r_a = a$. To satisfy the corresponding obedience constraint, we must have $\phi\pi_a^l(P-c) = (1-\phi)\pi_a^h c$.

Next, it is obvious that in problem $(\mathcal{P}')$ constraint (IR$_h$) is binding - otherwise we could reduce T_h without violating any other constraint. Furthermore, (A-IC$_l$) must be binding, because otherwise the first-best solution was implementable and optimal, which violates (A-IC$_l$). The constraint (IR$_l$) may or may not be binding.

As a last step, we have to show that the solution to problem ($\mathcal{P}'$) is also a solution to problem ($\mathcal{P}$). To show this, set $t_a = \theta_h q_a$ and $t_h = \theta_h q_h$. With these transfers, (IR$_h$) is kept unchanged. Setting

$$t_l = \theta_l q_l + \frac{\pi_a^h - \pi_a^l}{\pi_l^l}(\Delta\theta q_a - P) + \frac{\pi_h^h}{\pi_l^l}\Delta\theta q_h, \tag{23}$$

the remaining constraints (IC$_l$) and (IR$_l$) are satisfied. Obviously, all obedience constraints are satisfied. What remains to be shown is validity of (A-IC$_h$). To show this, we use the following two properties of the optimal mechanism, derived in Lemma 1 and Proposition 2: In any optimal mechanism $q_l = q_l^o$ and $q_h, q_a < q_l^o$. Furthermore, $\pi_a^h \geq \pi_a^l$.
Then the high-cost type's expected payoff from reporting θ_l is

$$\begin{aligned}
&\pi_l^l(t_l - \theta_h q_l) + \pi_a^l(t_a - \theta_h q_a)\\
&= -\pi_l^l \Delta\theta q_l + (\pi_a^h - \pi_a^l)\Delta\theta q_a + \pi_h^h \Delta\theta q_h - (\pi_a^h - \pi_a^l)P\\
&< -(\pi_l^l + \pi_a^l)\Delta\theta q_l + (\pi_a^h + \pi_h^h) q_l = 0
\end{aligned}$$

where the latter equality uses $1 = \pi_i^i + \pi_a^i$. Thus, (A-IC$_h$) is satisfied.

This completes the proof of Proposition 1. □

Proof of Lemma 1. Define

$$\begin{aligned}
\mathcal{W}(q_l, q_a, q_h) = &-\frac{(1-\phi)c}{P-c}\big(V(q_l) - \theta_l q_l - V(q_a) + \theta_l q_a\big) - \phi\Delta\theta(q_a - q_h)\\
&+ (1-\phi)\big(V(q_a) - \theta_h q_a - V(q_h) + \theta_h q_h\big) + \frac{\phi P - c}{P - c}P.
\end{aligned} \tag{24}$$

The Lemma states, that audits are beneficial if and only $\max_{q_l,q_a,q_h} \mathcal{W}(q_l, q_a, q_h) > 0$. Denote $(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) = \arg\max_{q_l,q_a,q_h} \mathcal{W}(q_l, q_a, q_h)$. Before continuing with the proof, we show the following properties of $\mathcal{W}$.

Lemma A.7. *1. For all $\varrho \in (0,1)$: $(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) = \arg\max_{q_l,q_a,q_h} \mathcal{V}(\varrho, q_l, q_a, q_h)$*

2. $(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) = (q_l^o, \widehat{q}_a, q_h^{na})$, where

$$V'(\widehat{q}_a) = \theta_h + \frac{\phi P - c}{(1-\phi)P}\Delta\theta \tag{25}$$

Proof. For the first point, differentiate $\mathcal{W}$, resp., $\mathcal{V}$, with respect to quantities.

$$\frac{\partial \mathcal{W}}{\partial q_l} = -\frac{(1-\phi)c}{P-c}(V'(q_l)-\theta_l) \qquad \frac{\partial \mathcal{V}}{\partial q_l} = \left(\phi - \varrho\frac{(1-\phi)c}{P-c}\right)(V'(q_l)-\theta_l) \tag{26}$$

$$\frac{\partial \mathcal{W}}{\partial q_a} = \frac{(1-\phi)P}{P-c}(V'(q_a)-\theta_h) + \frac{\phi P - c}{P-c}\Delta\theta \qquad \frac{\partial \mathcal{V}}{\partial q_a} = \varrho\frac{\partial \mathcal{W}}{\partial q_a} \tag{27}$$

$$\frac{\partial \mathcal{W}}{\partial q_h} = (1-\phi)(V'(q_h)-\theta_h) - \phi\Delta\theta \qquad \frac{\partial \mathcal{V}}{\partial q_h} = (1-\varrho)\frac{\partial \mathcal{W}}{\partial q_h} \tag{28}$$

Clearly, the right set of derivatives equals zero, if and only if the left set of derivatives does. Furthermore, (26) yields $q_l = q_l^o$, (28) yields $q_h = q_h^{na}$ and (33) follows from setting (27) equal to zero. □

Denote $\mathcal{V}^{na}$ the maximal profit the principal can achieve without auditing, i.e., the profit from the no-audit contract, and notice that $\mathcal{V}(\varrho, q_l, q_a, q_h) = \phi(V(q_l) - \theta_l q_l - \Delta\theta q_h) + (1-\phi)(V(q_l) - \theta_h q_h) + \varrho\mathcal{W}(q_l, q_a, q_h)$.

Now suppose $P < P^*$, which implies $\mathcal{W}(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) < 0$ by definition of P^*. Then, for all (ϱ, q_l, q_a, q_h)

$$\mathcal{V}(\varrho, q_l, q_a, q_h) \leq \mathcal{V}(\varrho, \widehat{q}_l, \widehat{q}_a, \widehat{q}_h) = \mathcal{V}^{na} + \varrho\mathcal{W}(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) < \mathcal{V}^{na},$$

and consequently the optimal mechanism entails $\varrho^* = 0$.

Next assume $P > P^*$ and define $\widehat{\varrho} := \sup\{\varrho \leq 1 | \varrho\Delta\theta\widehat{q}_a + (1-\varrho)\widehat{q}_h - \varrho P \geq 0\}$. Clearly, $\widehat{\varrho} > 0$, because $\widehat{q}_h > 0$. The quadrupel $(\widehat{\varrho}, \widehat{q}_l, \widehat{q}_a, \widehat{q}_h)$ is feasible in the constrained problem of maximizing (16) subject to (15). Thus,

$$\mathcal{V}^* \geq \mathcal{V}(\widehat{\varrho}, \widehat{q}_l, \widehat{q}_a, \widehat{q}_h) = \mathcal{V}^{na} + \widehat{\varrho} \cdot W(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) > \mathcal{V}^{na}.$$

Consequently, audits are strictly beneficial for $P > P^*$.

It remains to be shown that P^* exists and is unique. First, assume that $\phi P \leq c$. Because by assumption $P > c > 0$, (33) implies $\widehat{q}_l = q_l^o > \widehat{q}_a > q_h^{na} = \widehat{q}_h$. But then

$$\begin{aligned}
\mathcal{W}(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) &= -\frac{(1-\phi)c}{P-c}\big(V(q_l^o) - \theta_l q_l^o - V(\widehat{q}_a) + \theta_l\widehat{q}_a\big) - \phi\Delta\theta(\widehat{q}_a - q_h^{na}) \\
&\quad + (1-\phi)\big(V(\widehat{q}_a) - \theta_h\widehat{q}_a - V(q_h^{na}) + \theta_h q_h^{na}\big) + \frac{\phi P - c}{P-c}P \\
&< -\phi\Delta\theta(\widehat{q}_a - q_h^{na}) + (1-\phi)\big(V(\widehat{q}_a) - \theta_h\widehat{q}_a - V(q_h^{na}) + \theta_h q_h^{na}\big) \\
&\leq -\phi\Delta\theta(\widehat{q}_a - q_h^{na}) + (1-\phi)\frac{\phi}{1-\phi}\Delta\theta(\widehat{q}_a - q_h^{na}) = 0
\end{aligned}$$

where the weak inequality follows from the first-order condition for q_h^{na} and applying the mean-value theorem. Thus, whenever $\phi P \leq c$ we have $\mathcal{W}(\widehat{q}_l, \widehat{q}_a, \widehat{q}_h) < 0$. Next, differentiate $\max_{q_l,q_a,q_h} \mathcal{W}(q_l, q_a, q_h)$ with respect to P. Using the envelope-theorem and after some rearranging, this yields

$$\frac{(1-\phi)c}{(P-c)^2}\big(V(q_l^o) - \theta_l q_l^o - V(\widehat{q}_a) + \theta_l \widehat{q}_a\big) + \frac{\phi P - c}{P - c} + P\frac{(1-\phi)c}{(P-c)^2}.$$

Because we are only concerned with $\phi P > c$, the latter derivative is strictly positive. Hence, $\max_{q_l,q_a,q_h} \mathcal{W}(q_l, q_a, q_h)$ strictly increases with P for $\phi P > c$. This yields uniqueness of P^*. Existence follows from boundedness of $\widehat{q}_a$ and after observing that $P \cdot (\phi P - c)/(P - c)$ converges to infinity as $P \to \infty$. $\square$

Proof of Proposition 2. First assume (15) is slack. Using Lemma A.7 and Lemma 1, we have for $P > P^*$ and any $\varrho > 0$

$$\frac{\partial \max_{q_l,q_a,q_h} \mathcal{V}(\varrho, q_l, q_a, q_h)}{\partial \varrho} = \max_{q_l,q_a,q_h} \mathcal{W}(q_l, q_a, q_h) > 0. \tag{29}$$

Thus, the optimal mechanism has $\varrho^* = 1$. Furthermore, the mechanism corresponds to (RE), i.e. $q_l = q_l^o$ and $q_a = q_a^{RE}$. For this mechanism, (15) is satisfied if and only if $\Delta\theta q_a^{RE} \geq P$. Define $\underline{P}^m$ via

$$V'(\underline{P}^m/\Delta\theta) = \theta_h + \frac{\phi \underline{P}^m - c}{(1-\phi)\underline{P}^m}\Delta\theta. \tag{30}$$

Because $V(\cdot)$ is concave and $(\phi P - c)/P$ strictly increases, the value $\underline{P}^m$ is unique. The Inada-conditions on $V(\cdot)$ guarantee existence. Thus, whenever $P \leq \underline{P}^m$ the optimal mechanism is (RE) with (15) non-binding.

For the remainder assume $P > \underline{P}^m$ and hence (15) binding.
If $\varrho^* = 1$, then $q_a = P/\Delta\theta$ from (15). Maximizing the reduced profit function (which is independent of q_h) with respect to q_l yields $q_l = q_l^o$. Hence, this mechanism corresponds to (OA).

It remains to discuss the case where $\varrho^* \in (0, 1)$. To this end we solve the principal's problem with (15) binding. In case the solution entails $\varrho > 1$, the optimal mechanism then corresponds to (OA) from above.

Substituting $\varrho(q_a, q_h) := (\Delta\theta q_h)/(P - \Delta\theta(q_a - q_h))$ for ϱ, the principal maximizes

$$\phi\big(V(q_l) - \theta_l q_l - \Delta\theta q_h\big) + (1-\phi)\big(V(q_h) - \theta_h q_h\big) + \varrho(q_a, q_h)\mathcal{W}(q_l, q_a, q_h). \tag{31}$$

Furthermore, we have

$$\frac{\partial \varrho(q_a, q_h)}{\partial q_a} = \frac{\Delta\theta \varrho(q_a, q_h)}{P - \Delta\theta(q_a - q_h)}, \quad \frac{\partial \varrho(q_a, q_h)}{\partial q_h} = \frac{\Delta\theta(1 - \varrho(q_a, q_h))}{P - \Delta\theta(q_a - q_h)}. \tag{32}$$

Maximizing (31) with respect to quantities yields $q_l = q_l^o$ as well as the following first-order conditions

$$0 = \varrho(q_a, q_h)\left\{\frac{(1-\phi)P}{P-c}(V'(q_a)-\theta_h) - \frac{\phi P - c}{P-c}\Delta\theta\right\} + \frac{\Delta\theta\varrho(q_a,q_h)}{P-\Delta\theta(q_a-q_h)}\mathcal{W}(q_l^o, q_a, q_h), \quad (33)$$

$$0 = (1-\varrho(q_a,q_h))\left\{(1-\phi)(V'(q_h)-\theta_h) - \phi\Delta\theta\right\} + \frac{\Delta\theta(1-\varrho(q_a,q_h))}{P-\Delta\theta(q_a-q_h)}\mathcal{W}(q_l^o, q_a, q_h). \quad (34)$$

Because $P > P^*$ we have $\varrho > 0$, and together with (15) also $P - \Delta\theta(q_a - q_h) > 0$, in any solution to the principal's problem. Hence, (q_a, q_h) solve (33) and (34) if and only if they solve the following system of equations

$$0 = \big(P - \Delta\theta(q_a - q_h)\big)\left(\frac{(1-\phi)P}{P-c}(V'(q_a)-\theta_h) - \frac{\phi P - c}{P-c}\Delta\theta\right) + \Delta\theta\mathcal{W}(q_l^o, q_a, q_h), \quad (35)$$

$$0 = \big(P - \Delta\theta(q_a - q_h)\big)\big((1-\phi)(V'(q_h)-\theta_h) - \phi\Delta\theta\big) + \Delta\theta\mathcal{W}(q_l^o, q_a, q_h). \quad (36)$$

The right-hand side of (35) coincides with the right-hand side of (36) if and only if

$$V'(q_h) - \theta_h = \frac{P}{P-c}(V'(q_a)-\theta_h) + \frac{c}{P-c}\Delta\theta. \quad (37)$$

Hence, for any given q_a there exists a unique value $\widetilde{q}_h(q_a)$ solving (37). The two-dimensional system of equations given by (35) and (36) therefore reduces to the one-dimensional problem of finding a solution q_a such that the tuple $(q_a, \widetilde{q}_h(q_a))$ solves (35).

Next observe, that the solution cannot entail $q_a < \widehat{q}_a$. By (37) we have $\widetilde{q}_h(\widehat{q}_a) < q_h^{na}$. But then $\varrho q_a + (1-\varrho)q_h - \varrho P = 0$ implies $\varrho\widehat{q}_a + (1-\varrho)q_h^{na} - \varrho P > 0$ and hence $(\varrho, q_l^o, \widehat{q}_a, q_h^{na})$ is also feasible. Because $\mathcal{W}$ has the unique maximizer $(q_l^o, \widehat{q}_a, q_h^{na})$, we have $\mathcal{W}(q_l^o, q_a, q_h) < \mathcal{W}(q_l^o, \widehat{q}_a, q_h^{na})$. Similarly, $\phi(V(q_l^o) - \theta_l q_l^o - \Delta\theta q_h) + (1-\phi)(V(q_h) - \theta_h q_h) < \phi(V(q_l^o) - \theta_l q_l^o - \Delta\theta q_h^{na}) + (1-\phi)(V(q_h^{na}) - \theta_h q_h^{na})$. This implies $\mathcal{V}(\varrho, q_l^o, q_a, q_h) < \mathcal{V}(\varrho, q_l^o, \widehat{q}_a, q_h^{na})$, hence $(\varrho, q_l^o, q_a, q_h)$ was not optimal.

Also $q_a > q_h^o$ cannot be true for the optimal mechanism. To see this, notice first that by (37) we have $q_a > q_h$ if and only if $q_a < q_l^o$. Using this and the mean-value theorem we get

$$V(q_a) - \theta_h q_a - V(q_h) + \theta_h q_h < (V'(q_h) - \theta_h)(q_a - q_h) \leq \frac{c}{P-c}\Delta\theta(q_a - q_h). \quad (38)$$

Then also

$$\mathcal{W}(q_l^o, q_a, q_h) < -\phi\Delta\theta(q_a - q_h) + \frac{(1-\phi)c}{P-c}\Delta\theta(q_a - q_h) + \frac{\phi P - c}{P-c}P = \frac{\phi P - c}{P-c}(P - \Delta\theta(q_a - q_h)),$$

and thus

$$\begin{aligned}&\big(P-\Delta\theta(q_a-q_h)\big)\left(\frac{(1-\phi)P}{P-c}(V'(q_a)-\theta_h)-\frac{\phi P-c}{P-c}\Delta\theta\right)+\Delta\theta\mathcal{W}(q_l^o,q_a,q_h)\\ &<\big(P-\Delta\theta(q_a-q_h)\big)\frac{(1-\phi)P}{P-c}(V'(q_a)-\theta_h).\end{aligned}$$

Because in a solution with $\varrho^*\leq 1$ we must have $P-\Delta\theta(q_a-q_h)>0$ this yields a contradiction, because we assumed $q_a>q_h^o$ and by the above the right-hand side of (35) is strictly negative.

So far we have shown that any solution to the principal's problem must entail $\widehat{q}_a\leq q_a\leq q_h^o$. Evaluating (35) at $(\widehat{q}_a,q_h(\widehat{q}_a))=(\widehat{q}_a,q_h^{na})$ yields

$$\begin{aligned}&\big(P-\Delta\theta(\widehat{q}_a-q_h^{na})\big)\left(\frac{(1-\phi)P}{P-c}(V'(\widehat{q}_a)-\theta_h)-\frac{\phi P-c}{P-c}\Delta\theta\right)+\Delta\theta\mathcal{W}(q_l^o,\widehat{q}_a,q_h^{na})\\ &=\Delta\theta\mathcal{W}(q_l^o,\widehat{q}_a,q_h^{na})>0,\end{aligned}$$

because $P>P^*$. Similarly, at $(q_h^o,\widetilde{q}_h(q_h^o))$ we have

$$\begin{aligned}&\big(P-\Delta\theta(q_h^o-\widetilde{q}_h(q_h^o))\big)\left(\frac{(1-\phi)P}{P-c}(V'(q_h^o)-\theta_h)-\frac{\phi P-c}{P-c}\Delta\theta\right)+\Delta\theta\mathcal{W}(q_l^o,q_h^o,\widetilde{q}_h(q_h^o))\\ &<-\big(P-\Delta\theta(q_h^o-\widetilde{q}_h(q_h^o))\big)\frac{\phi P-c}{P-c}\Delta\theta+\frac{\phi P-c}{P-c}(P-\Delta\theta(q_h^o-\widetilde{q}_h(q_h^o)))=0,\end{aligned}$$

Hence, there exists a solution to (35) and (36), and this solution satisfies $\widehat{q}_a<q_a<q_h^o$.

To show uniqueness, differentiate the right-hand side of (35) with respect to q_a along (37). This yields

$$\begin{aligned}&(P-\Delta\theta(q_a-q_h))V''(q_a)-\Delta\theta\left(1-\frac{\partial q_h}{\partial q_a}\right)\left(\frac{(1-\phi)P}{P-c}\big(V'(q_a)-\theta_h\big)-\frac{\phi P-c}{P-c}\Delta\theta\right)\\ &\quad+\Delta\theta\left(\frac{(1-\phi)P}{P-c}\big(V'(q_a)-\theta_h\big)-\frac{\phi P-c}{P-c}\Delta\theta+\phi\Delta\theta\frac{\partial q_h}{\partial q_a}-(1-\phi)\big(V'(q_h)-\theta_h\big)\frac{\partial q_h}{\partial q_a}\right)\\ &=(P-\Delta\theta(q_a-q_h))V''(q_a)\\ &\quad+\Delta\theta\frac{\partial q_h}{\partial q_a}\left(\frac{(1-\phi)P}{P-c}\big(V'(q_a)-\theta_h\big)-\frac{\phi P-c}{P-c}\Delta\theta+\phi\Delta\theta-(1-\phi)\big(V'(q_h)-\theta_h\big)\right)\\ &=(P-\Delta\theta(q_a-q_h))V''(q_a)\end{aligned}$$

where the last equality uses (37). Because $P-\Delta\theta(q_a-q_h)>0$ this implies uniqueness of solution to (35) and, consequently, the system of equations (33) and (34) has a unique solution.
Provided this solution entails $\varrho^*<1$ it resembles the optimal mechanism and corresponds to (RE). Otherwise, we have $\varrho^*=1$ and mechanism (OA) as described above. □

Proof of Corollary 1. First, assume $\varrho^* = 1$. The incentive constraint of type θ_l is

$$\frac{\phi P - c}{\phi(P-c)}(t_l - \theta_l q_l) + \left(1 - \frac{\phi P - c}{\phi(P-c)}\right)(t_a - \theta_l q_a - P) = t_a - \theta_l q_a - P,$$

which implies

$$\frac{\phi P - c}{\phi(P-c)}(t_l - \theta_l q_l) = \frac{\phi P - c}{\phi(P-c)}(t_a - \theta_l q_a - P),$$

and thus $t_l - \theta_l q_l = t_a - \theta_l q_a - P$ because $(\phi P - c)/(\phi(P-c)) > 0$. Furthermore, the participation constraint implies $t_a - \theta_l q_a - P \geq 0$ and thus $t_l - \theta_l q_l \geq 0$.

Now assume the principal offers the agent the menu $\{(t_l, q_l, P_l(\cdot)), (t_a, q_a, P_a(\cdot))\}$, where $P_a(\theta_l) = P$ and all other penalties are zero. By the above derivations, type θ_l is indifferent between the two contracts, hence willing to randomize with the respective probabilities. Type θ_h prefers (t_a, q_a) and, provided the agent randomizes appropriately, the principal is indifferent between auditing and not after the agent did choose (t_a, q_a). It is in particular optimal to always audit in this case. The outcome trivially corresponds the outcome from employing the mediated mechanism.

Now assume $\varrho^* = 0$, then the optimal mechanism corresponds to the menu offer from the no audit contract.

Lastly assume $\varrho^* \in (0,1)$. In order to guarantee participation and randomization of type θ_h, we have to set $t_h = \theta_h q_h$ and $t_a = \theta_h q_a$. But then $t_a - \theta_l q_a - P < 0$, hence the low-cost type is not willing to randomize as in the mediated mechanism. Thus, the mechanism cannot be implemented with a menu offer. □