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Abstract

This paper studies the impact of private customer data about consumer naiveté in markets for deceptive products in which firms use these data to distinguish their existing customers' level of sophistication. To do so, I introduce a dynamic model in which competing firms can shroud hidden fees from naive customers, but not from sophisticated ones. Data on past usage is highly valuable to firms in competitive settings only if it identifies naive customers. Firms exploit private information on their existing customers' types to make type-specific offers. Since naives believe to be sophisticated, consumers do not self-select when given type-specific offers, making it impossible for rivals to compete effectively. Privately informed firms make offers to induce sophisticated customers to switch already at higher prices. Thus, competitors cannot attract profitable naives without attracting unprofitable sophisticates as well. This adverse-attraction effect enables firms to keep positive margins on existing naives, while breaking even on sophisticates. Since this implies that margins of naive consumers decrease in the share of sophisticated ones, firms prefer a balanced customer base. Achieving positive continuation profits from exploiting naive consumers requires each firm to have a substantial customer base. Thus, even when firms compete before learning about customers' types, firms have an incentive to coordinate on prices and competition is mitigated even more. I analyze the effects of a policy that discloses customer information to all firms and thereby increases consumer surplus, and illustrate the robustness of the results through several extensions.

Keywords: Deceptive Products; Shrouded Attributes; History-based Price Discrimination; Industry Dynamics; Big Data

JEL Codes: D14, D18, D21, D99, D89

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1 Introduction

Intuition as well as extensive empirical evidence suggests that in many markets firms understand consumer behavior or product features better than their customers do—such as credit cards (Ausubel (1991), Agarwal et al (2008), Stango, Zinman (2009, 2014)), retail banking (Cruickshank (2000), OFT (2008)), mortgages (Cruickshank (2000)), insurances (DellaVigna, Malmendier (2004)) or mobile phones (Grubb (2009))—allowing them to exploit consumer misunderstandings. I build on existing theoretical models studying such deceptive markets—in particular Gabaix, Laibson (2006) and Heidhues et al (2014)—when some consumers are naive, i.e. systematically underestimate their expenses for certain product components, and extend these static models to introduce dynamic competition. My model also captures a feature of increasing prevalence, namely that firms analyze their existing customers' data. By evaluating these data, firms have an informational advantage in identifying naive consumers relative to their competitors. I find that the rational model strongly underestimates the benefits of private consumption data to firms in competitive environments. Furthermore, if in addition firms can educate some consumers, my dynamic model highlights a novel competition impairing effect that leads to increased profits in a seemingly competitive market even before firms learn to distinguish their customers' level of sophistication.[2] Due to this effect, competition for market shares is mitigated despite the fact that they are valuable to firms.

Formally, I study a dynamic model with shrouded product attributes. N firms sell a homogeneous good in each period. They compete in prices for a unit mass of consumers to maximize discounted total profit. Naive customers do not take hidden fees into account but the sophisticates do and can avoid them.[3] Both types observe transparent fees. Firms first compete for market shares with symmetric information on customers but learn from their different purchase patterns to distinguish naive and sophisticated customers within their customer base; these information can then be used to price discriminate between existing customers based on their level of sophistication.

While there are in general many reasons for firms to collect data on their customers, I derive new insights on how firms benefit from collecting customer data when consumers differ in their

[2]I use the terms 'educate about' and 'unshroud' hidden fees interchangeably.

[3]This captures the idea that consumers can take precautions to avoid these fees. E.g. credit card owners can avoid interest payments by paying their debt in time. Mobile phone owners can avoid roaming charges by purchasing extra packages or calling from a land-line etc. But even without precautions, Heidhues et al (2014) show that sophisticated consumers that pay the 'hidden fee' can be screened into buying an alternative transparent product.

level of sophistication. Intuitively, when naive customers are unaware of hidden fees they pay, they accept the offer with the lowest transparent fee, just as sophisticated customers. Without the ability to identify customers' types, this makes it impossible to target sophisticates and naives separately. A customer's type can therefore not be inferred from his contract or product choice. Nonetheless, when firms can learn about their customers' naiveté by analyzing their purchase-history, this customer-type information allows firms to charge its clients differently according to their type. But since these heterogeneous clients do not self-select into competitors' offers, asymmetric information on naiveté create a competitive imbalance.

This introduces two novel effects into the literature on markets for deceptive products. First, when firms are privately informed about their clients' level of sophistication, i.e. via customer data, they can offer their sophisticates larger transparent prices than their naives. Firms thereby mitigate competition by inducing an *adverse-attraction effect*: competitors cannot attract a firm's profitable naive customers without attracting the unprofitable sophisticates as well. Firms make use of this and break even on their existing sophisticates while maintaining positive margins on their naives. They profitably exploit the fact that customer data allow them to discriminate between old customers while competitors lack the data to do so, and naive consumers lack the sophistication to recognize better offers. These results are independent of the ability of firms to educate consumers about hidden fees. Additionally, they imply that firms prefer a balanced customer base: as the share of naive customers increases, more customers pay a positive margin. But more naives also induce competitors to make better counter-offers, thereby decreasing the margin earned from naive ones.

Second, despite price competition with homogeneous products and initially symmetrically informed firms, ex post profits from private customer data might not be handed back to customers when firms compete for them in the first place. Firms without customer base earn zero profits and educate consumers about hidden fees to attract them. Thereby, they decrease profits for all competitors with a customer base. Consequently, the largest continuation profits can only be achieved when each firm has a substantial customer base. Firms, thus, want to make sure that their competitors get a sufficient portion of the market. This mitigates *competition for customer bases* by inducing firms to coordinate on prices. I establish in an extension that the same qualitative results hold when unshrouding is partial, i.e. when firms can only educate an arbitrarily small share of naives. This shows that transparency campaigns with the aim to educate consumers can serve as a credible threat to lower the profits of deceptive equilibria.

Natural policy suggestions in markets with deceptive products target consumer education or a simplified design of product offers. My results lead to the following alternative approach: disclose consumer data to all firms in the market. This allows competitors to approach profitable naive customers directly without attracting unprofitable sophisticates. Such a policy effectively splits the market, leading to marginal cost pricing for all customers and thus a sharp drop in firms' profits and an increase in consumer surplus. As a side effect, reduced profits increase incentives of firms to educate naive customers about hidden fees, rendering transparent pricing schemes more likely. An attractive feature of this policy is that it does not require regulatory knowledge about industry details or the ability to educate consumers. It also works if firms are unable to unshroud hidden fees. For example, when a credit-card company observes a profitable client of a competitor, it could try to inform the client about hidden fees, e.g. late-payment or overcharge fees he would save by switching. But if this fails, the company could offer him a lower transparent price instead that he takes into account, e.g. lower maintenance fees or a new-client bonus. In this way, disclosing customer information to competitors restores effective competition in cases in which private information on customer naiveté impede it.

One example of a market close to this setting is the one for credit cards. The product is quite homogeneous and maintenance fees, cash rewards, introductory APRs or new-client bonuses are usually taken into account. But many consumers ignore overlimit, overdraft or late fees or underestimate their tendency to borrow money when choosing a credit-card contract.[4] Firms condition offers on many observables and can learn to distinguish customers based on their naiveté.[5] Additionally, simple education policies of consumers w.r.t. hidden fees are effective.[6]

Schoar and Ru (2014) study the offers that credit-card companies make to customers with different (publicly) observable characteristics such as education or income. They find substantial variation in offers even after controlling for observable characteristics. Stango and Zinman (2013) study borrowing costs of credit-card customers and find that borrower risk and other observables explain only about half of the substantial dispersion in borrowing costs. These findings on price variation are in line with my result that firms play mixed strategies for their offers to new customers. They are also in line with firms conditioning offers on privately observed characteristics of their customers, i.e. from analyzing customer data. Additionally, Schoar and Ru (2014) find

[4]For empirical evidence, see Ausubel (1991), Agarwal et al (2008), Cruickshank (2000), OFT (2008), Stango and Zinman(2009,2014), Alan, Cemalcılar, Karlan and Zinman (2015).

[5]See Stango and Zinman (2009) or Grubb (2009). Alternatively, credit-card companies could estimate naiveté indirectly via their revealed elasticities for demand or by using big-data analysis on these elasticities.

[6]See Stango and Zinman (2014) or Alan, Cemalcılar, Karlan and Zinman (2015).

a larger price dispersion in subpopulations where consumers are more likely to be naive. This feature is predicted by my model as well.

Since the ability to distinguish naive and sophisticated customers is highly profitable even under perfect competition, my results suggest a new reason for why firms trade data on consumers. Consider, for example, data collected by internet search engines or loyalty programs such as those commonly used in retailing. In addition to analyzing consumption itself, big-data analysis is likely to improve firms' predictions about consumer naiveté. My findings therefore propose a new explanation for why firms benefit from big data, even when active in competitive markets.

Section 2 discusses the related literature. The basic setup is introduced in Section 3 and I discuss in more detail how the model captures crucial features of important markets such as credit-card markets, retail banking, markets for insurances or (mobile-)phone services.

Section 4 presents two benchmarks: i) A classical analogue with sophisticated consumers only, some of which purchase a base product while others also value an add-on. I show that when a firm can exclusively identify a customer's type and can condition offers on this information, it still earns zero equilibrium profits, since consumers choose offers optimally. ii) A benchmark in which firms do not learn to distinguish their existing customers' degree of sophistication, e.g. due to non-persistent types or customer data being uninformative about naiveté. Firms again earn zero equilibrium profits and there is cross-subsidization in total prices from naives to sophisticates.

Section 5 presents the main results discussed above on the profitability of private customer information on naiveté. Section 6 studies policy implications. In Section 7, I show robustness of the results to several extensions: (i) When naives cannot avoid hidden fees after being educated about them, firms can profitably attract their competitors' unavoiding naives by unshrouding. This imposes stronger existence conditions on shrouding equilibria but leaves them qualitatively unaffected. (ii) Partial unshrouding as discussed above. Further extensions are discussed in Appendix A and establish robustness concerning new customers arriving in period 2, learning of naive customers and $T > 2$ periods.

2 Related Literature

I discuss three areas of related literature. First, the literature on behavioral economics and exploitative contracting, second, the literature on adverse selection and worker poaching in labor markets, and third, the literature on customer poaching and switching costs.

Ausubel (1991) does an early empirical study of the credit-card market in the USA. Despite the market being highly competitive, he finds large profits. But while he suggests that search- and switching costs could explain parts of this profitability, he also states that these costs would need to be huge to explain the observed profits.[7] Though consumer misperceptions and misunderstandings seem an intuitive explanation for these observations, most papers that investigate these phenomena do not predict extraordinary profits under perfect competition: profits obtained from naive consumers are used to reduce transparent prices in order to attract more customers (e.g. see Gabaix and Laibson (2006), Armstrong and Vickers (2012) or Murooka (2013)).

The only paper I am aware of that offers an explanation for extraordinary profits in competitive environments based on misperception or misunderstanding is Heidhues, Kőszegi and Murooka (2014). They study profitable deception and inferior products. Deception is profitable due to a price floor on transparent fees, which prevents firms from handing over all profits from hidden fees to consumers via reduced transparent prices. This price floor is motivated by adverse selection, i.e. firms try to distract unprofitable consumers whose valuation is below marginal costs, or suspicion, where consumers infer from very low prices that a good must be "bad" while higher prices allow for the possibility of a "good" product. They establish that firms can share a common interest in guaranteeing each firm a certain market share to keep shrouding stable. This, however, does not lead to increased overall shrouding profits. Their framework is close to mine, but I study a dynamic model where firms can learn to distinguish their old clients based on their sophistication and I do not impose a price floor. In retail-banking or credit-card applications, a price floor may not apply, since a minimal required product use (e.g. minimal monthly cash-inflows on a bank account) can be and often is required by banks. This allows banks to attract customers while making sure that very unprofitable types are not attracted. My paper offers an alternative explanation for extraordinary profits in competitive environments that is based on the firms' ability to use their customer data to distinguish clients based on their sophistication. I also show that the firms' common interest in keeping their competitors' market shares sufficiently large can have important competition-impairing effects and lead to increased total shrouding profits.

Gabaix and Laibson (2006) introduce a model where firms sell a transparent base good with an add-on price that is shroudable and not taken into account by naive consumers. Sophisti-

[7]In particular, per outstanding balance of 1,000\$, monthly search and switching costs in the range of 250\$ were required to explain the observed profits.

cated consumers take hidden add-on fees into account and can avoid them by taking costly and inefficient steps in advance. Their main contribution is to show that in some market equilibria, firms do not want to unshroud hidden fees to consumers since profitable naive consumers become sophisticated and can therefore not be attracted in a profitable way. Building on that model and applying it to the UK retail-banking industry, Vickers and Armstrong (2012) analyze a model with contingent charges. They argue that the existence of naive customers can explain the common "free-if-in-credit" model that charges nothing for account maintenance but contingent charges like overdraft fees or interest payments to generate revenues.

I build on these models and extend them to a dynamic setting in which firms' information on consumer naiveté plays a role. My results, however, differ in some crucial aspects: in my model, shrouding increases equilibrium profits and cross subsidization between customer types is limited. Additionally, shrouding occurs even if there is only a small share of naive consumers. Moreover, firms prefer a mix of naive and sophisticated customers to only naives.

Other papers study the role of information of firms with respect to consumers' level of sophistication. Heidhues and Kőszegi (2014) consider seller information on customer naiveté and establish that third-degree price discrimination can lower welfare when firms discriminate based on naiveté. But in contrast to my model, they consider symmetric information of firms on the consumers' degree of sophistication. Though these symmetric information on naiveté have important welfare implications, they do not explain extraordinary profits. Kamenica, Mullainathan, Thaler (2011) study asymmetric understanding of firms and consumers. They discuss the impacts of a regulation that informs consumers about their own behavior and thereby reduces this asymmetry. Such a regulation can increase consumer welfare when firms keep prices constant. But when prices are adjusted, consumer and producer surplus remain unchanged. My approach is different in that I consider information that help firms to identify customers based on their sophistication. This allows me to study the impact of price discrimination and the role of asymmetric information about customers on *competition* between firms. Consequently, I suggest a disclosure policy that does not rely on better informed consumers but rather on better informed competitors. I show that this eliminates the competitive advantage that firms have due to their superior knowledge about old consumers, and increases consumer welfare.

Murooka (2013) analyses the incentives of intermediaries to sell a deceptive product rather than a transparent one. Because intermediaries earn high commissions despite competition by selling deceptive rather than transparent products, shrouding equilibria exist in which only the deceptive product is sold. These shrouding equilibria can be eliminated by regulating commissions. This induces intermediaries to reveal hidden attributes to consumers.

A crucial feature of this paper is that naive and sophisticated customers are considered that cannot be screened ex ante because naives falsely believe to be sophisticated. Eliaz and Spiegler (2006) discuss screening of adversely-naive agents in a setting where all agents are ex-post identical but differ in their beliefs about their realized ex-post type. In contrast, I study agents with different ex-post types but identical ex-ante beliefs.

The adverse-attraction effect established here is reminiscent of adverse selection and worker poaching in the labor market literature. Greenwald (1986) and Gibbons and Katz (1991) study labor markets where a worker's productivity is observed by the current employer. Competing employers try to poach workers from other firms, but they neither know their productivity nor can they offer contingent contracts. In these models, employers let their low-productive workers go to other firms but keep their high-productive ones. Results crucially depend on the assumption that firms cannot make offers that are contingent on the firms ex-post observable private information, i.e. the workers' productivity. Gibbons and Katz (1991) justify this assumption by stating that contracts contingent on productivity of workers can often not be enforced (i.e. productivity might be observable, but it is not verifiable towards third parties like a court). Otherwise, if productivity was verifiable, poaching firms could make offers that promise a bonus in case a high productivity is observed and a punishment if productivity is low. This would attract only high productivity workers and adverse selection would not be observed in equilibrium. In fact, Mirrlees (1974) and Riordan and Sappington (1987) show that with ex-post public signals, even in monopoly settings first-best outcomes can be implemented quite generally. I emphasize this point because verifiability of consumption of additional goods or services is given in many consumer markets: a bank can easily verify whether a customer overdrew on an account and phone companies can verify the number of calls from customers to any phone number and contracts that specify prices for each of these events are standard practice. Therefore, one would expect adverse-selection effects similar to worker poaching to be less important when the asymmetric information of firms is not workers' productivity but their customers' demand for additional goods or services. In contrast, I find that adverse selection can be important even in consumer markets with demand-related verifiable private information when some consumers are naive.

Deceptive products might seem reminiscent of products with switching costs.[8] But there are

[8]For a detailed survey on switching costs, see Farrel, Klemperer (2007).

some fundamental differences. Markups due to switching costs are efficient when they prevent consumers from switching too often, whereas markups via hidden fees exploit naiveté. Additionally, basic dynamic models with switching costs exhibit a bargain-then-ripoff structure. Firms with small market shares price aggressively to attract customers while larger firms set higher prices to profit from their customers' switching costs. In my model, though, while prices are also high for existing customers, they are high for new customers as well.

The literature of switching costs also considers history-based price discrimination as in Chen (1997) and Taylor (2003). Chen (1997) studies a model with two periods and two firms selling a homogeneous product. After the first period, consumers draw uniformly distributed switching costs. Firms price discriminate between old and new customers and earn profits from both. This prevents firms from competing aggressively in the first period and induces positive total profits. Taylor (2003) extends this framework to more than two firms. With three or more firms, the second-period monopoly in attracting customers vanishes and total profits go to zero. In contrast, I predict positive profits even with competition for existing customers.

Stole (2007), summarizes another strand of the literature on history-based price discrimination that focuses on customer poaching. The poaching literature studies price discrimination between old and new customers with horizontally differentiated products.[9] A common finding is that even a monopolist does not benefit from history-based price discrimination. Forward-looking consumers take higher future prices into account and benefit from not consuming in the first period to get a lower price in the second one. In contrast, I find that firms benefit from price discrimination between their old customers even under perfect competition. Additionally, forward looking sophisticates cannot do better by not purchasing in the first period.

Note that both in the switching cost and poaching literature, firms usually discriminate between own and competitors' customers but not between different types of own customers. This discrimination, however, is a crucial feature of my analysis.

3 The Basic Model

Before continuing with the basic model, I introduce the concept of deceptive markets and common applications. Readers who are familiar with the literature might want to skip this subsection.

[9] See Fudenberg, Villas-Boas (2006) for a survey.

3.1 Applications

The goal of models on deceptive products is to understand markets where consumers do not take some product characteristics into account, because they misperceive product- or pricing features, or misestimate their own future demand at the time of contracting/purchase. The reduced form model analyzed in this paper covers both cases. Markets where a potential for deception has been established empirically include those for credit cards, insurances, mortgages, retail banking, (mobile-)phone services, printers and casinos. The additional feature of my model in relation to the literature is repeated consumption of the product combined with the firms' ability to infer the level of sophistication of their customers by observing past behavior. Potential applications for this model are markets for credit cards, retail banking, (mobile-)phone services and insurances, since both deception and behavior/history based pricing occur and information collection of customers' purchase patterns is simple and pervasive.[10]

Credit cards are a quite homogeneous product and mainly vary in fee structures. Many consumers do not take overlimit, overdraft or late fees into account or underestimate their tendency to borrow money when choosing a credit-card contract.[11] In my model, every such fee that is not taken into account by naives is represented by the hidden fee. Prices that are taken into account, such as maintenance fees, cash rewards, introductory APRs or new-client bonuses, are transparent fees.[12] For examples on how firms can learn to distinguish customers based on their naiveté, see Stango and Zinman (2009) or Grubb (2009).[13] Alternatively, credit-card providers could measure naiveté indirectly by simply estimating a consumer's elasticities of demand for the different fees and check how these elasticities correlate with the clients' purchase patterns. Towards this goal, firms can additionally use big-data analysis on the purchase details of customers or the usage-patterns on online accounts.

Besides this, some simple education policies of consumers w.r.t. hidden fees are effective, as shown by Stango and Zinman (2014). They observe that simply asking consumers about overdraft

[10] In all of these markets, detailed information on consumption patterns are required to write bills to customers.

[11] For empirical evidence, see Ausubel (1991), Agarwal et al (2008), Cruickshank (2000), OFT (2008), Stango and Zinman(2009,2014), Alan, Cemalcılar, Karlan and Zinman (2015). Heidhues et al. (2014) also discuss the application of the model to credit card markets. Consumers pay additional hidden fees because of their taste for immediate gratification that leads them to borrow more than they prefer ex ante. See also Meier and Sprenger 2010 for a discussion of existing evidence.

[12] Though heterogeneity within these two blocks of fees is certainly important, it is not the focus of this analysis.

[13] Stango and Zinman (2009) identify hidden fees by the savings in fees that a credit-card customer could have made by shifting liquidity between accounts. Grubb (2009) looks at how much telephone customers could have saved if they had the same consumption with a different contract. Though the latter approach might not distinguish customers who are naive from those purchasing a commitment device, it probably provides an informative signal, which is enough for the purpose of this paper.

fees in a survey significantly reduces their probability of paying those fees relative to a control group that was not asked these questions. Similarly, Alan, Cemalcılar, Karlan and Zinman (2015) inform some customers of a Turkish retail bank of the possibility of overdraft without mentioning prices while others are offered a discount. While mentioning overdraft without mentioning prices increases use of overdraft, offering a discount reduces it. This strongly suggests that overdraft prices are indeed shrouded to customers and that simple information campaigns can be effective in unshrouding hidden components to at least some customers

In retail banking, empirical evidence suggests that customers underestimate their likelihood of overdraft when choosing a bank-account (see OFT (2008) or Cruickshank (2000)). Hence, fees and interest payments associated with overdraft are hidden fees to many consumers. Account maintenance fees are rather salient and more likely to be transparent fees.

The (mobile-)phone market is studied by Grubb (2009). At the time of contracting, firms have better forecasts on consumers' later demand for phone calls; i.e. when consumers underestimate the variation of their demand, firms can offer contracts with high payments in states that customers falsely perceive as unlikely. These unexpected payments also function as hidden fees.

Applications to insurance markets work in a similar way. Given the huge amount of data that insurance companies have over their customers, they are likely to have better estimates on at least some of their clients' risks than these have themselves. This could lead to customers paying more for their insurances than they would if they knew their true risks.

As these applications show, fees in observed contracts do not need to fit perfectly into the categories hidden or transparent fee. A more general way to think about transparent and hidden fees is as anticipated and unanticipated payments. E.g. take a credit-card customer. Assume he pays 10€ maintenance fees and 0.10€ for each Euro he does not pay back within 30 days. Say he believes he will borrow 50€ for more than 30 days while he will actually borrow 100€. Then in this model his transparent fee is 10€ + 5€ = 15€ and his hidden fee the unanticipated 5€.

3.2 Setup

There are two periods. Firms sell a homogeneous product in each period to a unit mass of consumers. In each period consumers value consuming the product at $v > 0$ and their outside option at zero. There are two types of consumers. The share $1 - \alpha \in [0, 1]$ is sophisticated. They observe transparent and hidden price components, and can avoid paying the hidden one at no costs.[14] The share α is naive and takes only transparent prices into account when firms shroud hidden fees. Naive consumers who are educated about hidden fees become sophisticated, i.e. when a firm unshrouds hidden prices, naives take them into account for all firms and can avoid them. In extensions I relax these assumptions and discuss partial education and naives who can not avoid unshrouded hidden fees. Consumers maximize their perceived utility.[15]

Naive consumers are assumed not to learn about their naiveté over time, except when educated by a firm. This is consistent with empirical evidence of consumers triggering fees they are unaware of repeatedly but—as I show in extensions—relaxing this simplifying assumption does not change results qualitatively.[16] I assume that consumers, once educated about hidden fees, remain so for the subsequent period.

There are $N \geq 2$ firms, each with marginal cost $c \geq 0$. In each period t, firm n sets hidden and transparent price components a_{nt} and f_{nt}. The set of customers who purchased from a firm in period 1 are called its customer base. Firms learn their customers' types by observing their consumption patterns in period 1, i.e. by noting that naives pay the hidden fees and sophisticates do not. Since firms alone observe the consumption of their own customers, customer data are private information to each firm. Firms can charge different prices to different consumers when they can identify their types. In period 2, this enables each firm n to charge f_{n2}^{naive} and f_{n2}^{soph} to naive and sophisticated consumers in its customer base, respectively. In order to attract new customers from competitors, firm n charges a new-customers price denoted f_{n2}^{new}. Since naives believe to be sophisticates and hence select contracts identically, a single price to attract the two customer types is not restrictive. In period 1, not knowing any consumer's type, firms only set one price f_{n1}. In each period, firms set a hidden fee $a_{nt} \in [0, \bar{a}],\ t = 1, 2.$[17] I follow the literature by assuming a price cap $\bar{a}$ on hidden prices.[18] Additionally, firms choose whether to educate

[14]In the credit-card example, many consumers pay overcharge fees or interests on their credit-balance despite having liquid funds available to reduce their balance to avoid these fees (see Stango and Zinman (2009)). Thus, for sophisticated consumers with liquid funds, a simple money-transfer avoids hidden fees while naives make no use of this option. Note that sophisticates that cannot avoid hidden fees, can be screened by firms into purchasing a transparent non-deceptive product, which is why I do not consider them. For details on this, see Heidhues et al. (2014).

[15]Within each period, naive consumers perceived utility of purchasing from firm n is $v - f_n$, while their actual utility is $v - f_n - a_n$. They correctly perceive the former when unshrouding occurs. Sophisticated consumers' perceived and actual utility of purchasing from n is $v - f_n$. In period 1, consumers take their perceived continuation utility into account.

[16]See Cruickshank (2000), Stango and Zinman (2009,2014) or OFT (2008).

[17]Not conditioning a_{n2} on observed types is w.l.o.g. since sophisticates avoid them and naives do not. Making an offer to sophisticates that has no hidden fee leads to the same payments as an offer with a hidden fee.

[18]Gabaix, Laibson (2006) and Armstrong, Vickers (2012) make the same assumption. They justify the assumption by legal or regulatory restrictions on fees or consumers only noticing a fee when it is sufficiently large, i.e. above such a threshold. Another way to think about $\bar{a}$ is as the maximal willingness to pay for an additional service that consumers require unexpectedly after signing the contract.

consumers about hidden prices in each period.[19]

When consumers are indifferent between all firms, I employ a general tie-breaking rule: each firm gets a market share $s_n > 0$ with $\sum_{n=1}^{N} s_n = 1$. When indifferent between less than N firms, I impose for ease of exposition that market shares are assigned proportionally.

Sorting Assumption: Among firms that make them indifferent, consumers are sorted independently of their type. This simplifies the analysis by guaranteeing that—given shrouding— the distribution of types within a non-empty customer base is the same as in the overall population.

The timing of the game is as follows:

Period 1: Competition for a Customer Base

- *Firms* simultaneously choose transparent prices f_{n1} and hidden fees a_{n1}, and decide whether to educate consumers about hidden fees or not.

- *Consumers* buy from the firm they perceive as the cheapest, given it is preferred to their outside option. Hence if consumers are not educated about hidden fees, both types choose a firm n, where $n \in \text{argmax}_{n' \in N}\, v - f_{n'1} + V_{n'2}$, where $V_{n'2}$ denotes the expected continuation utility in period 2 after consuming from firm n' in period 1, while naive types additionally pay $a_{n'1}$. When a firm unshrouds hidden prices, naive consumers become sophisticates.

Period 2: Asymmetric Information on the Firms' Customer Bases

- After observing which of their customers in period 1 payed the hidden fee, firms learn their old customers' types. Customer-base information is private: a firm can only identify the type of its own customers.

- *Firms* choose a price to the sophisticated and naive consumers in their customer base, denoted f_{n2}^{soph} and f_{n2}^{naive}, respectively. Additionally, they can set a price to attract customers from competitors, denoted f_{n2}^{new}. Firms choose hidden fees a_{n2} and whether to educate consumers about hidden fees or not.

- *Consumers* purchase from the firm they perceive to be the cheapest. If hidden fees are shrouded, a consumer of type $\theta \in \{soph, naive\}$ who purchased from firm n in period 1 picks the smallest price in $\{f_{n2}^{\theta}, (f_{\hat{n}2}^{new})_{\hat{n} \neq n}\}$ conditional on this price being smaller than v. When hidden prices are unshrouded, naives become sophisticated and solve the same problem but without paying hidden fees.

[19]Formally, firm n chooses $e_{nt} \in \{0, 1\}$, where $e_{nt} = 1$ means firm n educates consumers in period t about hidden fees. Unshrouding is costless in the basic model but in the extensions I discuss the implications of positive unshrouding costs.

I apply the concept of Perfect Bayesian Equilibrium. Despite firms having beliefs on the composition of their competitors' customer bases, PBE is relatively straightforward here since the Sorting Assumption pins down beliefs: after shrouding occurs in period 1, all firms with a non-empty customer base have the same type distribution in their customer bases. With unshrouding in period 1, all customers become identical and type information and beliefs are obsolete. Hence, beliefs on the composition of the competitors' customer bases only matter after shrouding in period 1 and are then identical to the distribution of types in the population. This is why I do not point out beliefs explicitly throughout the paper and focus on sequential rationality.

In what follows, I study the existence and properties of shrouding equilibria, that is, equilibria in which shrouding occurs with positive probability. In addition, Bertrand equilibria always exist where at least two firms unshroud hidden fees and all consumers pay marginal cost for consuming the product. Since those are less interesting and arguably less robust, I do not focus on them throughout the paper.[20]

4 Benchmarks

To emphasize the impact of consumer naiveté and private customer data, I analyze two benchmark cases. First, a classical analogue where all consumers are perfectly sophisticated and value a base product, but only some consumers value an add-on as well. For example, some consumers only buy a credit-card account to do transactions while others also borrow. Afterwards, I study the basic model absent customer data, i.e. where firms do not learn their old customers' types. In both benchmarks, profits are zero and expected consumer surplus is maximized.

4.1 Private Customer-Base Information without Naive Consumers

In this benchmark all consumers value the base good with $v > c$, but the share α of consumers—called *add*—buy an add-on good for which they have valuation $\bar{a}$. The remaining consumers who only buy the base good are called *base*. There are two firms A and B, which produce the base good at cost c and the add-on without additional marginal costs. W.l.o.g., let firm A know all customers' types while firm B knows only their distribution.[21] Thus, firm A can assign prices for each type, f_A^{add}, f_A^{base}, while B can instead offer two products—the base product only and a

[20]Heidhues et.al. (2014) argue that Bertrand equilibria are not reasonable in the context of deceptive products. Among other things, they are not robust to positive unshrouding costs—no matter how small.

[21]Generally, the same argument holds w.r.t. all consumers of which A knows the types and B does not since firms can set different prices for customers in- and outside of their customer base.

product with add-on—at different prices f_B^{add}, f_B^{base}.

A simple screening argument shows that firm A cannot benefit from her information. To see this for pure strategies, first note that firm B cannot earn positive margins from any customer type. Otherwise, firm A—being able to target each customer group—could increase profits by marginally undercutting prices for each customer group. Now suppose towards a contradiction that firm A earns a positive margin from any customer group. Suppose A profitably offers $f_A^{add} > c$ to types *add*. Then firm B can earn strictly positive profits by setting $f_B^{add} = f_A^{add} - \epsilon$ and $f_B^{base} = f_A^{add} + \epsilon$ for some $\epsilon > 0$ small enough. *add* consumers self select into paying f_B^{add}, *base* consumers either stay with A or profitably self select into paying f_B^{base} and B earns strictly positive profits—a contradiction. Similarly, we cannot have $f_A^{base} > c$, since B could then profitably draw all *base*s by setting $f_B^{base} = f_A^{base} - \epsilon$ and $f_B^{add} = f_A^{base} - 2\epsilon$ for some $\epsilon > 0$ small enough. This leads to a contradiction as well. These results are extended to mixed strategies in the following proposition.

Proposition 1. [Private Customer Information with Sophisticated Consumers only]
When customers are sophisticated and have heterogeneous add-on demand, a firm that is privately informed about add-on-demand types earns zero profits from each type in a competitive market.

In competitive markets, firms offer first-best contracts. When consumers are sophisticated, they make optimal choices and self-select into the first-best contract. Thereby, they reveal their information by their product choice such that in competitive environments, private information of firms on willingness to pay for add-ons is unprofitable.

In the credit-card context this means that after correcting for non-demand heterogeneity such as risk levels of customers etc., profits from consumers that borrow with their credit-card account and from those that simply use their credit card for transactions should be similar. This prediction on margin levels extends to environments with switching or search cost, as long as these do not asymmetrically differ across the two customer groups.

Remark: Of course, also in models where all consumers are sophisticated, firms can have many reasons to gather information on their customers that are beyond the scope of this paper. But as shown in Section 5, the rational model strongly underestimates the benefits of information on customers if some consumers are naive.

4.2 No Customer Data

The next benchmark highlights the role of customer data in the presence of naive customers. I look at the two-period model for deceptive products when there are naive and sophisticated consumers but firms do not learn their customers' types.

When customers cannot be distinguished, firms offer only one transparent price in each period. Discrimination between own and competitors' customers does not help since they follow the same distribution and have the same preferences. Results are summarized in the following proposition.

Proposition 2. [Deceptive Markets without Customer Data]
Let $v \geq c - \alpha\bar{a}$.[22] *Shrouding equilibria exist. In each shrouding equilibrium, firms earn zero profits. In each equilibrium in which shrouding occurs with probability one, consumers pay transparent prices $f_{n1} = f_{n2} = c - \alpha\bar{a}$ and naives additionally pay hidden prices $a_{n1} = a_{n2} = \bar{a}$.*

Proposition 2 translates the results of Gabaix and Laibson (2006) to this setting. The main difference is that there are two periods but—when firms are unable to distinguish customers by naiveté to price discriminate in period 2—there are no dynamic effects. The equilibrium is simply a repetition of the one-period equilibrium discussed by Gabaix and Laibson (2006), in which there is cross-subsidization of total prices from naive to sophisticated consumers within each period.

The intuition is as follows: given shrouding, hidden fees increase margins without affecting consumers' decisions and are consequently set to $\bar{a}$. But since firms cannot price discriminate between consumers, they use profits from hidden fees to lower transparent prices and attract consumers until the average customer is not profitable anymore. This reduces profits to zero in both periods in this benchmark.

Shrouding equilibria are not very stable since firms earn the same profits by unshrouding prices. In particular if some naive customers cannot avoid unshrouded hidden fees, as shown in Section 7.1, they can be profitably attracted by unshrouding hidden fees so that a shrouding equilibrium never exists for socially valuable products.

For future reference, note also that the continuation profits on any equilibrium path are zero in the second period, whether hidden fees are shrouded in the first period or not.

[22] The assumption guarantees that sophisticated consumers want to buy in equilibrium even when the product is socially wasteful. For the case of $v < c - \alpha\bar{a}$, firms do not sell to sophisticates anymore. This has been studied by Heidhues et al. (2012).

The two benchmarks establish that if either all consumers are sophisticated or firms do not have private data on the usage histories of consumers, profits are zero and there is inefficient trade in the latter case if $c - \alpha\bar{a} < v < c$. The next section studies the main model introduced in Section 3.2.

5 The Benefits of Customer Data in Deceptive Markets

I now discuss the model introduced in Section 3. Before looking at the propositions, I illustrate why firms earn positive profits in the second period of shrouding equilibria and why prices are random in equilibrium. Proposition 3 establishes how firms benefit from informational advantages in distinguishing their old customers. Proposition 4 discusses why these profits might not be handed over to customers in period 1.

Consider period 2 after prices are shrouded in period 1 and all firms have a positive customer base. Firms set two different prices for their own old customers, f_{n2}^{naive} and f_{n2}^{soph}, and one to attract customers from competitors, f_{n2}^{new}. Note that since firms can identify their old customers, firms do not attract their own sophisticates with new-customer prices below c.

To see that period 2 profits are positive in a shrouding equilibrium, consider the simple case with two firms A and B and suppose both firms shroud hidden fees. Look at firm A and note first that $f_{A2}^{soph} \geq c$ since firm A would otherwise prefer not to sell to old sophisticated customers. This implies that B always attracts sophisticates of A with prices $f_{B2}^{new} < c$. Since naive customers pay hidden fees of $\bar{a}$ in each shrouding equilibrium, only $f_{B2}^{new} \geq c - \alpha\bar{a}$ can lead to non-negative profits for B from attracting new customers. All $f_{B2}^{new} < c - \alpha\bar{a}$ induce strictly negative profits for B from new customers. Thus, roughly speaking, prices $f_{A2}^{soph} < c$ and $f_{B2}^{new} < c - \alpha\bar{a}$ cannot occur in an equilibrium with positive probability. But with $f_{B2}^{new} \geq c - \alpha\bar{a}$, there is no reason for firm A to price its naives below $c - \alpha\bar{a}$. This implies $f_{A2}^{naive} \geq c - \alpha\bar{a}$. The same reasoning holds for firm B and can be generalized to any number $N \geq 2$ of firms (see Figure 1). Consequently, firms can always deviate to achieve profits of at least $s_n\alpha(1-\alpha)\bar{a}$ from consumers in its customer base by setting $f_{n2}^{naive} = c - \alpha\bar{a}$ and $f_{n2}^{soph} \geq c$. Setting $f_{n2}^{new} \geq c$ ensures that these profits are not wasted by unprofitably attracting new customers. Though this is not an equilibrium, it establishes the minimum profits firms can guarantee themselves in each shrouding equilibrium

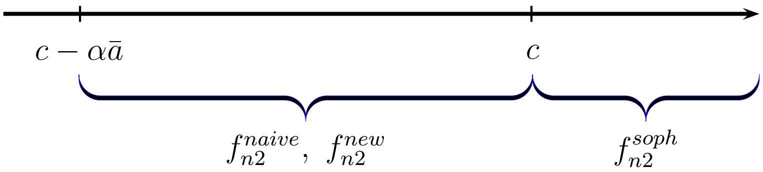

Figure 1: Support of prices in period 2. Firms only keep sophisticated consumers if they at least break even with them. Thus, there is no cross-subsidization. New customers' prices are above $c - \alpha\bar{a}$, implying no need to price naive customers below this threshold.

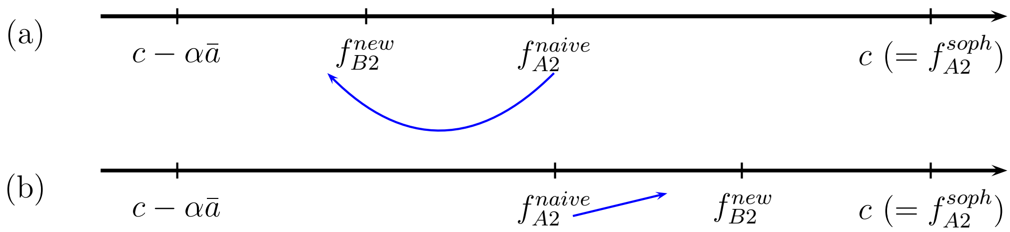

Figure 2: No pure strategy equilibrium in period 2: (a) Firm A could profitably keep her naive customers by undercutting f_{B2}^{new}. (b) Firm A could further increase profits by moving f_{A2}^{naive} closer to f_{B2}^{new}. Hence, both situations are no equilibria.

in period 2.

Next, let us see why there is no pure-strategy shrouding equilibrium in period 2. Take again two firms A and B and let $f_{A2}^{naive} > c - \alpha\bar{a}$. By marginally undercutting f_{A2}^{naive} with f_{B2}^{new}, firm B can profitably attract A's customers. Firm A can prevent this by charging $f_{A2}^{naive} = c - \alpha\bar{a}$. Then firm B attracts no naive consumer from A and charges some $f_{B2}^{new} \geq c$ to break even on new customers. But then, A is better off by increasing her naive-customer price to $f_{A2}^{naive} = c$. This, however, gives B an incentive to marginally undercut $f_{A2}^{naive} = c$ and the argument starts again (see Figure 2).

In the end, after shrouding in period 1 and when all firms have a positive customer base, consumers pay transparent naive and new-customer prices based on the following distributions:

$$F^{new}(f_n^{new}) = \begin{cases} 0, & \text{if} f_n^{new} \in (-\infty, c - \alpha\bar{a}] \\ 1 - \sqrt[N-1]{\frac{(1-\alpha)\bar{a}}{f_n^{new}+\bar{a}-c}}, & \text{if} f_n^{new} \in (c - \alpha\bar{a}, c) \\ 1, & \text{if} f_n^{new} \in [c, \infty) \end{cases}, \quad \forall n. \tag{1}$$

Observe that the distribution has a mass point of weight $\sqrt[N-1]{1-\alpha}$ on c.

$$F^{naive}(f_n^{naive}) = \begin{cases} 0, & \text{if} f_n^{new} \in (-\infty, c - \alpha\bar{a}] \\ \frac{f_n^{naive}+\alpha\bar{a}-c}{\alpha(f_n^{naive}+\bar{a}-c)}, & \text{if} f_n^{new} \in (c - \alpha\bar{a}, c) \\ 1, & \text{if} f_n^{new} \in [c, \infty) \end{cases}, \quad \forall n. \tag{2}$$

With these mixed strategies at hand, Proposition 3 summarizes the results for period 2.

Proposition 3. [Exploiting Private Information on Customer Data in Period 2]
Consider any equilibrium in which shrouding occurs with positive probability in the second period. Then, shrouding occurs with positive probability in period 2 if and only if hidden prices are shrouded in period 1 and each firm has a non-empty customer base.

In such equilibria, profits are $\pi_{n2} = s_n\alpha(1-\alpha)\bar{a}$. Hidden prices are shrouded with probability one and $a_{n2} = \bar{a}$. Transparent prices are $f_{n2}^{soph} \geq c$ and consumers pay f_{n2}^{new} and f_{n2}^{naive} based on (1) *and* (2) *respectively.*

The supports of the distributions show that information advantages w.r.t. customer bases create an *information-based price floor* in the second period of shrouding equilibria at $c - \alpha\bar{a}$. Firms earn positive margins on their old naives and break even on sophisticates and new customers. Note that on $(c - \alpha\bar{a}, c)$ the distributions are symmetric and identical in each second period of shrouding equilibria. Intuitively, all firms $j \neq n$ mix new-customer prices to make firm n indifferent between all $f_n^{naive} \in (c - \alpha\bar{a}, c)$. This must be true for all n and therefore all new-customer prices must follow the same distribution. The same logic applies to naive-customer price distributions. There are other equilibria where $F^{new}(\cdot)$ has no mass point on c or $f_{n2}^{soph} \geq c$. But as shown in detail in the proof of Proposition 3, all shrouding equilibria lead to the same profits, consumers' purchase prices, and welfare.

The *adverse-attraction* of unprofitable sophisticated customers creates an information-based price floor at $c - \alpha\bar{a}$ for naives. Private information on customer data allow firms to price their old customers differently based on their sophistication. But since naives believe to be sophisticated, competitors can only attract new customers with a single offer. This creates a competitive asymmetry. Because firms keep their old sophisticates only if they are profitable, firms charge them at least marginal-cost prices. At the same time, transparent prices of naive customers are below marginal costs. Thus, firms use their private information on naiveté to render unprofitable sophisticated customers more responsive to new offers than profitable naive ones. Attracted profitable naive customers are always accompanied by some unprofitable sophisticates but sophisticates might be attracted without naives. This mitigates the competitive pressure on naive-customers' margins and these margins remain strictly positive. Since firms earn zero profits by attracting new customers on average, overall profits are strictly positive.

Importantly, the properties of these shrouding continuation equilibria do not depend on the ability of firms to unshroud hidden fees and hence these insights carry over to cases where consumer education is infeasible or very costly.

The above proposition sheds new light on the value of customer data in competitive environments. To see how, compare the results with Propositions 1 and 2. When consumers are aware of their demand for additional services, they optimally respond to counter offers. Proposition 1 shows that this renders customer data unprofitable when consumers are sophisticated but differ in their demand for an add-on. When there are naive consumers but firms cannot distinguish customers' types, naives pay more but these profits are handed over to sophisticated customers, inducing cross subsidization. Firms earn zero profits and the benefits from exploitation end up with sophisticated consumers. In contrast, firms are able to keep the revenue from exploitation when they learn to distinguish customers based on their naiveté. To see that this has strong effects on profits, note that the overall market revenue from hidden fees in period 2 is $\alpha\bar{a}$. Of this amount, firms manage to keep the share $(1-\alpha)$ despite competition. For example, if $\alpha = 0.5$, half of the hidden fees payed remain as profits to firms. Thus, Proposition 3 establishes the main result of the paper. Namely that firms benefit strongly from their customer data by being able to distinguish naive and sophisticated customers.

Another interesting finding is that shrouding profits are not monotone in the share of naive consumers α. Firms earn a positive expected margin of $(1-\alpha)\bar{a}$ from naives, but this margin is decreasing in the share of naives. Due to the adverse-attraction effect, a larger share of sophisticates makes it less profitable to compete fiercely for a competitor's customers. Thus, firms can keep a larger margin on its naives. This has two important implications: First, firms might want to educate some customers about hidden fees but not too many. Second, a common finding in the literature is that shrouding conditions require a sufficiently large amount of naive consumers.[23] But when firms can distinguish customers based on their naiveté, shrouding equilibria can exist with arbitrarily small shares of naive customers in the population. This extends to the case in Section 7.1 where unavoiding naives make unshrouding more profitable.

Also note that each firm strictly prefers shrouding over unshrouding. In particular, since naives become sophisticated after unshrouding and can then avoid hidden fees, the most profitable deviation by unshrouding hidden prices is the same as in Proposition 2 and leads to zero profits. But in contrast to Proposition 2, firms earn positive profits here, giving them a strict incentive to keep prices shrouded when they learn to distinguish the naiveté of their customers. Thus, customer data make shrouding more stable.

In line with the mixed strategies for new-customer prices, Schoar and Ru (2014) find that credit-card companies have substantial variations in their offers to new consumers, even after controlling for available observable characteristics. Similarly, Stango and Zinman (2014) observe substantial variation in borrowing costs for credit-card customers after controlling for observable characteristics. Both findings are in line with the mixed strategies in Proposition 3 and the firms conditioning on privately observed characteristics. Also in line with the supports of (1) and (2), Schoar and Ru (2014) find a larger price dispersion for subpopulations where consumers are more likely to be naive, i.e. have a lover level of education.

Before looking at period 1, I discuss equilibrium selection.

Equilibrium Selection. Continuation profits are zero whenever unshrouding occurs in period 1 or at least one firm has an empty customer base. The latter is true since firms without customer base have no naive customers to exploit and earn zero profits. But when all naives can avoid unshrouded hidden fees, unshrouding induces zero profits as well and firms without customer base are indifferent between unshrouding or not. Thus, there is multiplicity of equilibria. But Proposition 6 below shows that whenever there is a positive share of naives that cannot avoid hidden fees after unshrouding, the multiplicity disappears and when at least one firm has an empty customer base, unshrouding occurs with probability one.[24]

Intuitively, unavoiding consumers can be profitably attracted after unshrouding since they are

[23]Shrouding conditions makes sure that no profitable unshrouding deviation exists. For examples, consider Gabaix, Laibson (2006) or Murooka(2013).

[24]The existence of unavoiding naives is particularly realistic in the credit-card example: consumers that become aware of high overdraft fees and interest rates still have some amount of credit-card debt. In order to avoid those fees, they require other liquid assets to pay back their whole credit-card debt which is impossible for consumers with liquidity constraints.

aware of hidden fees but still pay them. While educated avoiding naives need to be attracted by undercutting transparent prices, unavoiding naives can be attracted by marginally undercutting their total price, i.e. transparent price plus hidden fee. Firms without a customer base then have a strict incentive to educate customers. This allows me plausibly to focus on equilibria in which firms without customer base educate consumers with probability one, since other equilibria are not robust to the presence of unavoiding naives.

This also shows that positive total profits do not depend on the multiplicity of stage-game equilibria in finitely repeated games and a related collusion-type logic. And since unavoiding naives make unshrouding strictly profitable for firms without a customer base, results are also robust to positive costs for unshrouding.

Bertrand equilibria exist next to the shrouding equilibrium where at least two firms unshroud hidden fees and all consumers pay a total price of marginal costs. Therefore, I make an *equilibrium selection assumption*: Whenever a shrouding equilibrium exists in period 2, firms will play it. This is plausible, especially since the shrouding continuation equilibrium is strictly preferred by each firm over the Bertrand one.[25]

We saw that customer data on naiveté can be profitably exploited by competing firms. I show now how competition for customer bases is mitigated as well when firms can educate consumers about hidden fees. In the main text I maintain the assumption that unshrouding reaches all naive customers to keep the exposition simple. In an extension I verify that results are robust to partial unshrouding that reaches only an arbitrarily small share of consumers.

Denote by $s_{min} = \min_n\{s_n\}$ the smallest market share and the set of all firms that charge the lowest price in period 1 by $M = \{n \in \{1, 2, ..., N\} | f_{n1} = \min_n\{f_{n1}\}\}$. Then we can write down the total profits given firms shroud in period 1 (Figure 3):

$$\pi_{n1}(f_{11}, ..., f_{N1}) = \begin{cases} \frac{s_n}{\sum_{o \in M} s_o}(f_{n1} + \alpha\bar{a} - c) + 0, & \text{if } f_{n1} = \min_{n'}\{f_{n'1}\} \leq v \ \& \ M < N \\ s_n(f_{n1} + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a}, & \text{if } f_{n1} = \min_{n'}\{f_{n'1}\} \leq v \ \& \ M = N \\ 0, & \text{if } f_{n1} > \min\{v, \min_{n'}\{f_{n'1}\}\} \end{cases} \quad (3)$$

Total profits exhibit a new kind of discontinuity that stems from the dynamic nature of the game and the possibility to educate consumers about hidden fees. Positive continuation profits

[25] Heidhues et.al (2014) argue that this is the only reasonable equilibrium. Among other things, the Bertrand-type equilibrium is not robust to positive unshrouding costs.

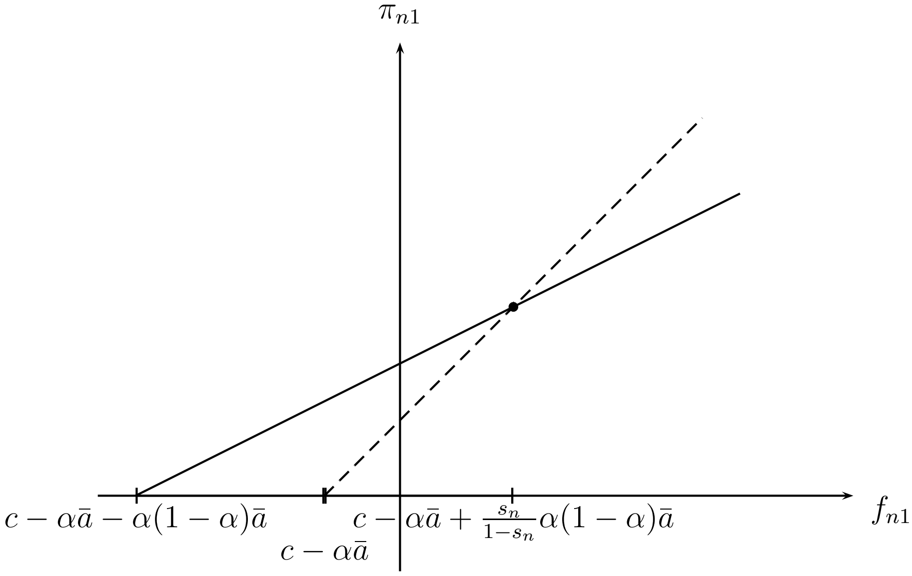

Figure 3: The dotted line depicts total profits of a firm that undercuts all others in period 1. The solid line depicts total profits of a firm when all firms choose the same price in period 1. Hence, for all $f_1 \in [c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}]$, no firm has an incentive to undercut competitors.

can only be achieved when prices are shrouded in period 1 and all firms attract customers in this period. That is, all firms charge $min_n\{f_{n1}\}$ with positive probability. This results in a strong incentive to coordinate on the same transparent price.

Proposition 4. [Mitigated Customer-Base Competition in Period 1 in Shrouding Equilibria] *Shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying the selection criteria, all firms choose hidden fees $a_{n1} = \bar{a}$. In equilibria with pure strategies in period 1, all firms set the same transparent price $f_1 \in \left[c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$. Total profits $\Pi_n = s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \in \left[0, s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}\right]$. For all equilibria in which $\Pi_n > 0$, shrouding occurs with probability one.*

Profitable shrouding in period 2 can occur only if all firms have a positive customer base. Therefore firms benefit from coordinating prices in period 1. If this coordination fails, some competitors are left without a customer base and continuation profits are zero since firms without customer base can only attract customers by educating naives. This *mitigates customer-base competition* already in the first period. Future profits are not competed away ex ante, but instead total profits can increase above the second period level.

First-period transparent prices are not uniquely pinned down. Firms have an incentive to coordinate on a whole interval of prices. The coordination incentive also gives rise to—arguably less plausible—mixed-strategy equilibria with some miscoordination. I discuss them in the appendix

In all but one shrouding equilibrium, firms earn strictly positive total profits. Since unshrouding leads to zero profits, firms have a strict preference to shroud given all others do. In comparison to the benchmark cases, shrouding is not only more profitable, but also more stable when firms can distinguish their customers level of sophistication. Note that when some naive consumers cannot avoid unshrouded hidden fees, under reasonable conditions, firms earn strictly positive total profits in each shrouding equilibrium. This renders shrouding equilibria even more stable and suggests that total profits might be positive even when changes in demand are more smooth than with Bertrand competition. For more on this see Section 7.1.

From the firms' perspective, shrouding equilibria with higher transparent prices in period 1 Pareto dominate equilibria with lower prices. This gives firms an incentive to coordinate on higher transparent prices in the first period. The following corollary summarizes the results of Propositions 3 and 4 when Pareto dominance is applied as an equilibrium-selection device.

Corollary 1 [The Firms' Preferred Shrouding Equilibrium]
In the most profitable shrouding equilibrium, firms charge $f_{n1} = c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a},\ \forall n$ *and second-period prices are as in Proposition 3. Hidden fees are* $a_{nt} = \bar{a}$ *in both periods and total profits are* $\Pi_n = s_n \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}$*. Shrouding occurs with probability one.*

The comparisons with Propositions 1 and 2 show that private information on old customers have a strong impact on the properties of shrouding equilibria. Total prices increase in both periods for all customer types. Shrouding becomes more stable since profits can be positive in each period. These results are driven mainly by two effects: Second-period shrouding profits increase due to the *adverse-attraction effect* in Proposition 3. *Mitigated competition for customer bases* explains why these profits might not be handed over to consumers in the first period despite the profitability of market shares in shrouding equilibria.

The model highlights new and important dynamic effects in markets for deceptive products. While there are no dynamic effects in Proposition 2, they become crucial when firms learn about their customers. The competition for the market in period 1 works very different from competition within the market in period 2, and the results differ in crucial aspects from usual properties of markets for deceptive products: information on customer naiveté become a valuable asset for firms. Additionally, the results suggest that firms have a reason not to become informed about competitors' customers since this would intensify competition and decrease shrouding profits.

Note that the high profits of firms in the second period of shrouding equilibria do not depend on the firms ability to educate consumers about hidden fees, but positive profits in the first period do. Though such transparency policies are possible, they are very unlikely to reach all naive consumers. Extensions with an imperfect unshrouding technology are discussed in Section 7.2 and lead to the same qualitative results.

Before discussing policy implications, note that despite concerns about safety-in-markets or consumer surplus, efficiency can be a concern as well. In a richer model with a smoothly decreasing demand curve, the price distortions away from marginal costs induce overconsumption of naive customers as well as for sophisticated ones.[26] At the same time, large profits could lead to inefficient investments in exploitative technologies or excessive entry.[27] The policies discussed below could improve efficient consumption in such a framework, by moving prices closer to marginal costs.

A natural policy suggestion derives from the results above: firms should disclose their private information on their customers to their competitors, i.e. their customer data.[28] The impacts of such a policy are discussed in the following section.

6 Policy Implications

Consumption data are usually not only accessible by firms but also by consumers. Especially since firms are required to write a bill to consumers—phone bills depend on how much and which network was called, credit-card bills depend on payments made with the card and the resulting overall balance—the consumption data are in principle available to consumers as well and can therefore be given to competing firms.

In the context of this model, disclosing consumer data to all firms enables each firm to charge different prices to each customer type, whether it is in the firms' customer base or not. The impact of such a policy is summarized in the next proposition.

Proposition 5. [Deceptive Markets with Disclosed Customer Info in Period 2]

[26] For more on this, see Heidhues, Kőszegi (2014).

[27] This is discussed by Heidhues, Kőszegi and Murooka (2014).

[28] Thaler and Sunstein (2008) discuss a policy that aims at simplifying consumer data and make them available easily to consumers in order to help them make better decisions. In contrast to this, the policy discussed below aims at sharing customer information with competing firms, not customers.

Firms earn zero profits in each second-period continuation equilibrium and in period 1. Equilibria exist where shrouding occurs with probability one. In these equilibria, consumers pay total prices equal to marginal costs in period 2 and transparent prices $c - \alpha\bar{a}$ in period 1. Hidden prices are $a_{n1} = a_{n2} = \bar{a}$. If shrouding does not occur with probability one in period 2, it occurs with probability zero.

First of all, note that deviating by unshrouding makes firms earn zero profits as well. Thus, the firms' incentives to unshroud hidden prices become stronger relative to the case in Propositions 3 and 4.

With consumers' types disclosed to all firms in period 2, the market is effectively split and firms compete for each customer type separately. This induces marginal cost pricing even in shrouding equilibria and zero profits. Thereby, the disclosure policy triggers a rent shift from firms to consumers.

Besides the regulatory benefits, Proposition 5 highlights together with Proposition 2 that indeed the asymmetric information on customer data cause high profits of firms when consumers are naive.

This policy increases consumer surplus also when firms are unable to educate consumers. To see this, consider any second period of the game with shrouding in period 1. Absent unshrouding abilities for the firms, this case covers all histories. Then the logic behind Proposition 5 says that profits are competed away, even when consumers are not educated about hidden fees. The extension with partial unshrouding strengthens the importance of this observation even more: when firms can only educate some consumers about hidden fees, the remaining naives can still be exploited, leaving positive profits to firms even after unshrouding. With customer data disclosed to all firms, however, profits still go to zero.

A particularly nice feature of this policy is that it is not based on educating customers or providing them with tools that help them to make better decisions. Policies that go in this direction are discussed by Sunstein and Thaler (2008) and Kamenica, Mullainathan and Thaler (2011). Despite the fact that empirical findings strongly suggest that consumers are not aware of product or contract features in some markets, it is not always clear how exactly those features are misunderstood. Before inducing an effective simplification or education policy, we would have to understand first the psychological process underlying consumers' misunderstandings. Thus, such policies require deep regulatory knowledge, a feature they share with well-designed price-setting interventions. In contrast, disclosing customer data to competitors is much less sensitive to regulatory knowledge. The policy simply limits the firms' abilities to profitably price discriminate customers, since competitors can now target each customer group specifically.

Note that banning price discrimination is beneficial to consumer surplus as well and leads to the same outcomes as described in Proposition 2. But banning price discrimination in credit-card or retail-banking markets would probably have many unintended consequences. Especially since discriminating consumers along other dimensions, e.g. their risk behavior, is likely to increase welfare.

7 Extensions

7.1 Unavoiding Naives

After unshrouding hidden fees, naive consumers could either be able to avoid them and become like sophisticates, or become aware of hidden fees without being able to avoid them. Both assumptions can be reasonable. As an example, consider the availability of external funds for credit-card or retail-bank borrowing. Consumers with external funds can easily avoid costs of borrowing by paying back debt immediately. Consumers with liquidity constraints cannot. [29]

Assume that a share $\eta \in [0, 1)$ of the naives cannot avoid unshrouded hidden fees, though they take them into account. The remaining $1 - \eta$ naives can avoid them and become like sophisticated consumers after unshrouding.[30]

The qualitative properties of shrouding equilibria do not change when (some) naive consumers cannot avoid unshrouded hidden fees. This is because both types are identical when hidden fees are shrouded. But the incentives to unshroud hidden fees change and thus, the existence of shrouding equilibria becomes an issue. When all firms shroud hidden fees, unavoiding naive consumers pay a total prices above marginal cost. When now a firm unshrouds hidden fees unavoiding naives still pay the hidden fee and can therefore be profitably attracted. This renders shrouding conditions more restrictive. In particular, unshrouding and marginally undercutting the smallest total price of shrouding competitors for their naive customers attracts all unavoiding naives and makes them pay $\min\{c + (1 - \alpha)\bar{a}, v\}$. Overall, this gives $\alpha\eta \min\{(1 - \alpha)\bar{a}, v - c\}$ as

[29] Alternative examples are situations where consumers can avoid hidden fees by precautionary behavior that is not available in the short term. Expensive roaming charges can be partially avoided by booking additional packages or purchasing a local phone-card. But when an urgent phone call has to be made, those preparations cannot be done quickly. Note that sophisticated consumers that cannot avoid hidden fees can be screened into another product as in Heidhues et.al. (2014), so I do not consider these types here.

[30] The case $\eta = 1$ is ruled out to avoid that firms are indifferent between shrouding or not when only considering their own customer base.

profits of deviating from a shrouding equilibrium by educating customers in period 2. The following proposition summarizes the general existence conditions of shrouding equilibria for the results in Sections 4 to 6.

Proposition 6. [Shrouding Conditions with Unavoiding Naives]
Assume the share $\eta \in [0,1)$ of naive consumers cannot avoid unshrouded hidden fees while the others can avoid them costlessly.

1. *When firms do not learn their customers' types, the shrouding equilibrium as in Proposition 2 exists if and only if*

$$0 \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v-c\}. \tag{4}$$

   When this condition is violated, unshdrouding occurs with probability one.

2. *When firms learn their customers' types, shrouding continuation equilibria as in Propositions 3 exist if and only if*

$$s_n\alpha(1-\alpha)\bar{a} \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v-c\}, \quad \forall n. \tag{5}$$

   When this condition is violated for at least one firm, unshdrouding occurs with probability one in the first period. If (5) holds, shrouding equilibria with shrouding in both periods exist.

   If total prices are below v, i.e. $f_1 + \bar{a} \leq v$, in each pure-strategy equilibrium, all firms charge $f_1 \in c - \alpha\bar{a} + \left[-\alpha(1-\alpha)\bar{a}, \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$, giving rise to total profits $\Pi_n = s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \in \left[0, s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}\right]$. Shrouding occurs with probability one.

   If total prices are above v, i.e. $f_1 + \bar{a} > v$, in each pure-strategy equilibrium, all firms charge $f_1 \in c - \alpha\bar{a} + \left[\frac{\eta\alpha}{s_{min}}(v-c) - \alpha(1-\alpha)\bar{a}, \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$, giving rise to total profits $\Pi_n = s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \in \left[\eta\alpha(v-c), s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}\right]$. Shrouding occurs with probability one.

3. *The results of Proposition 5 remain unchanged.*

Note that for $\eta = 0$, the deviation profits of unshrouding become zero and we are in the special case depicted in Section 5. Again, there are mixed-strategy equilibria for first-period prices which are discussed in the appendix.

The most important result of this section is that in the presence of unavoiding naives, firms with an empty customer base have a strict incentive to unshroud hidden prices. This motivates the equilibrium selection discussed before Proposition 4 and implies that results are robust to positive unshrouding costs.

Another interesting result of Proposition 6 is that in case 2, for $\eta > 0$, total profits are strictly positive in each shrouding equilibrium when $f_1 + \bar{a} > v$.[31] Intuitively, since unavoiding naives can be profitably attracted, unshrouding is now a deviation strategy that leads to strictly positive profits. In these deviations, firms undercut total prices to attract unavoiding naives and set transparent prices at marginal cost to at least break even with sophisticated and educated avoiding naive customers. Now distinguish two cases.

First, if $f_1+\bar{a} > v$, unshrouding firms can maximally extract the total valuation of unavoiding naive consumers when all other firms shroud. Therefore maximal profits from unshrouding are $\eta\alpha(v-c)$ which is independent of f_1. But then transparent prices when shrouding occurs must be large enough to earn at least $\eta\alpha(v-c)$. If firms do not earn at least these constant unshrouding profits, a firm will unshroud with probability one. As a result, total profits must also be strictly positive.

Second, if $f_1+\bar{a} \leq v$, unshrouding firms can maximally earn $\eta\alpha(f_1+\bar{a}-c)$ when all competitors shroud. This is not a constant but depends itself on f_1. When this price level is low enough, i.e. for firms to earn zero total profits, unshrouding leads to negative profits while shrouding is still profitable. Thus, shrouding equilibria with lower prices can be maintained.

When customer information of firms on naiveté is symmetric—i.e. the cases of Proposition 2 and 5 when types are not persistent, data are not informative on naiveté or naiveté is fully disclosed—Proposition 6 shows that shrouding equilibria either earn zero profits or do not exist. They do not exist for socially beneficial products ($v \geq c$) and even if $v < c$, they do not exist whenever $\eta > 0$. When customer-base information is private, however, positive profits induce the existence of shrouding equilibria. Thus, the results on the incentives to educate consumers become even sharper in the presence of unavoiding naives: firms have an incentive to shroud hidden fees only when firms can distinguish customers based on their sophistication.

Remark: If I allow firms to offer multiple contracts to customers, there would be additional equilibria. In particular, firms could make unshrouding unprofitable to competitors for any value

[31] This condition is particularly interesting as it always holds at the margin in a richer setup in which consumers valuations are continuously distributed.

of η by offering an additional product to their existing naive customers with $(\hat{f}_{n2}, \hat{a}_{n2}) = (c, 0)$. If shrouding occurs, no consumer prefers this product to the one she gets in the equilibrium discussed in Proposition 3. But if unshrouding occurs in period 2, educated unavoiding naive consumers are better off by choosing $(\hat{f}_{n2}, \hat{a}_{n2})$ instead of switching to a competitor. Thus, in the second period and for any $\eta > 0$ firms with zero market shares are indifferent between unshrouding or not. In addition to the shrouding equilibria in Propositions 3 and 4, this would induce additional equilibria in which firms always shroud in the second period, i.e. when they have an empty customer base, and total profits are zero. This reasoning, however, relies on the fact that $(\hat{f}_{n2}, \hat{a}_{n2}) = (c, 0)$ is never chosen on the equilibrium path. It is, thus, not robust to consumers wrongly choosing this contract. To illustrate this, suppose naive consumers of firm n wrongly choose this contract with probability $\epsilon > 0$. Since these naives would also pay a hidden fee, firm n is strictly better off by increasing $\hat{a}_{n2}$ to $\bar{a}$. As a consequence, naives of firm n pay a total price above c such that competitors of n can unshroud hidden fees and profitably attract these unavoiding naive consumers. This argument shows that such offers that render unshrouding unprofitable are not robust to being chosen by mistake.

In the rest of the paper, I return to the case $\eta = 0$ to simplify the exposition.

7.2 Partial Unshrouding

When firms start a transparency campaign or simplify their pricing scheme to make consumers aware of hidden fees, they will probably not affect all naive consumers. At the same time, such policies are unlikely to be without any effect at all. Examples for the effectiveness of simple interventions are given by Stango and Zinman (2014) and Alan, Cemalcılar, Karlan and Zinman (2015) and are discussed in more detail in Section 3. In this section, I show that the main results of Propositions 3 and 4 are robust to partial unshrouding. To this end, I assume that after unshrouding, only the share $\lambda \in (0, 1]$ of naives takes hidden fees into account.

Proposition 7. [Partial Unshrouding]

- *For $\lambda \in (0, 1]$, shrouding equilibria with shrouding in period 2 exist if and only if hidden prices are shrouded in period 1 and each firm has a non-empty customer base. Prices and profits under shrouding are as in Proposition 3.*

- *Shrouding equilibria with shrouding in both periods exist. In each equilibrium satisfying the selection criteria of Proposition 4, all firms choose hidden fees $a_{n1} = \bar{a}$. In equilibria with* *pure strategies in period 1, each firm sets the same transparent price* $f_1 \in \left[c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} - \frac{1-\lambda}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$*. Total profits are* $\Pi_n \in \left[0, s_n[\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} - \frac{1-\lambda}{1-s_{min}}\alpha(1-\alpha)\bar{a}] + s_n\alpha(1-\alpha)\bar{a}\right]$*. For all these equilibria in which* $\Pi_n > 0$*, shrouding occurs with probability one.*

The first bullet point states the robustness of second period of shrouding equilibria for the case of partial unshrouding. Naives are unchanged when shrouding occurs, which is why the properties of shrouding equilibria in period 2 remain unaffected. Since some consumers remain naive firms could now earn positive profits conditional on unshrouding in period 2, but these profits are strictly smaller than profits conditional on shrouding.

Intuitively, observing a customer's naiveté after period 1 is less informative when unshrouding occurs in period 2 since some old naives turn sophisticated. Thus, firms award transparent fees below marginal cost also to old naive customers who are sophisticated after unshrouding and can now avoid hidden fees. This renders customer data less profitable after unshrouding in period 2.

The second bullet point states that for each $\lambda > 0$, shrouding remains both possible and profitable in period 1. At first, this seems to contradict the earlier observation that firms have a preference for a balanced customer base. If there are many naives and unshrouding reaches only some consumers, unshrouding would result in a more balanced customer base. So why do firms not want to unshroud in period 1? Firms deviating from a shrouding equilibrium in period 1 can either undercut competitors while hidden fees remain shrouded or unshroud hidden fees and attract competitors' customers. The proof shows that they prefer the former. Intuitively, firms face the following trade-off:

First, when competitors' customers are attracted in period 1, the deviating firm earns hidden fees of $\bar{a}$ from each attracted naive customer. Thus, direct earnings from hidden fees are larger in period 1 when firms simply undercut competitors without unshrouding prices since more consumers remain naive. In period 2, however, hidden fees will be unshrouded in both cases. But a firm that already unshrouded in period 1 has better information on which customers remain naive in period 2. Therefore, a firm that unshrouds and undercuts in period 1 can keep a larger margin in the second period. But since this margin is only $(1-(1-\lambda)\alpha)\bar{a} < (\bar{a})$, this benefit from better information and a more balanced customer base in period 2 is smaller than the direct decrease of earnings from hidden fees in period 1.

Undercutting competitors in period 1 while keeping prices shrouded is therefore the most profitable deviation from the shrouding equilibrium path for all λ. Firms, therefore, have no increased incentive to unshroud in period 1 under partial unshrouding, even though this could increase their future profits by giving them a more balanced customer base. This is because educating customers today comes with a direct loss in margins today, which is larger than the increase in margins tomorrow.

Undercutting competitors in period 1 becomes more profitable under partial unshrouding since some naives remain in the market that can be exploited in period 2.[32] Thus, deviations from shrouding equilibria in period 1 now have positive continuation profits. Deviations from period 1 equilibrium prices are therefore more beneficial and the firms' Pareto-dominated equilibrium price becomes smaller. This reduces the largest total profits that firms can achieve. But even though total profits shrink, they remain positive for all $\lambda > 0$, leaving results qualitatively unchanged.

Note also that the effects of the disclosure policy are unchanged and all profits are competed to zero, though some consumers will always remain naive.

7.3 Consumer Learning and T Periods

I discuss further extensions in Appendix A: when new customers arrive in period 2 or when some naive customers learn about hidden fees after period 1, results do not change qualitatively. In both cases, observing naiveté in period 1 remains an informative signal on naiveté in the second period such that firms earn positive expected profits from old naive customers while breaking even on all others. In another extension I look at a model with two firms and T periods. I establish that shrouding equilibria exist where shrouding occurs with probability one in each period. Intuitively, firms benefit from not learning to distinguish their competitors' customers because this induces firms to compete more aggressively on naive consumers and reduces continuation profits to zero.

8 Conclusion

This paper investigates the role of customer data in markets in which firms can employ their customers' consumption data to predict the likelihood of customer mistakes. While customer data can also be valuable in rational models, my results suggest that the rational model severely

[32] Note that firms face the same coordination issue as inherent in Proposition 4. Firms can generate positive continuation profits by setting the same price with positive probability. Equilibria with mixed first-period transparent prices exist as well but are discussed in the appendix. In each mixed equilibrium, firms set the same finite number of transparent prices so that these equilibria exhibit the same coordination incentive as the equilibria with pure strategies in period 1.

underestimates the firms' benefits of private customer data. This paper, therefore, gives a novel explanation for high profits—excluding any fixed cost of operation—in seemingly competitive markets such as the credit-card industry.

Beyond consumption data, another informative type of consumer data is big-data analysis. It is frequently used to help firms to better predict their customers' behavior. In particular when big-data analysis allows firms to predict their customers' degree of sophistication, the results of this paper shed new light on the role of big data in competitive markets. When firms manage to get hold of their customers usage data, e.g. via cookies, search histories, or by requiring them to create an online account that facilitates the observation of usage patterns, firms can gather a lot of usage data that can be used to distinguish customers based on their naiveté. This paper therefore offers a new explanation on how big data related to such services or search engines can be profitably used or sold even to firms active in competitive markets. As my benchmarks show, this is not obvious without naive consumers since sophisticated consumers optimally self-select into efficient offers made by competing firms.

In addition, big-data analysis has the potential to introduce a novel form of asymmetric understanding to market settings when consumers are unaware of the informational traces they leave behind. Shiller (2014), for example, finds that the number of websites visited on Tuesdays and Thursdays predict demand for netflix accounts while surfing on a Wednesday seems to carry little information. He also simulates that netflix could have raised profits by only 0.8 percent when using price discrimination based on usual demographic characteristics. By using data on browsing behavior, such as website visits on Tuesdays and Thursdays, profits could have been increased by 13 percent. It is hard to imagine that many consumers take the effect on prices into account when browsing the internet.

I show that disclosing consumer data to competitors can be beneficial in breaking the competitive asymmetry created by the interaction of consumer naiveté and private information thereof. Even if this policy would not trigger firms to offer more transparent products, it reduces transparent prices to naive consumers. When implementing such a policy, however, consumers might be concerned about their data privacy. But since consumers with the same characteristics are offered the same (expected) price, firms could be given data about customers in an anonymous way without affecting the results of such a policy. A potential drawback is that in a richer model with a heterogeneous participation decision of consumers, this policy can lead to excessive participation in the market by naive consumers. This policy might decrease transparent prices to naive customers, and when these prices are below marginal cost, naives might overparticipate in the market.[33] But at the same time, lower profits reduce incentives of firms for excessive entry and to inefficiently invest in exploitative innovation.

In many of the industries that involve naive consumers, another crucial dimension of heterogeneity is riskiness of consumers. In consumer borrowing, consumers usually differ in their likelihood of paying their debt and in insurance markets, consumers have different levels of risk against which they want to be insured. The analysis in this paper can be viewed as conditional on a realization of such a risky dimension. Combining the two heterogeneities is left for future research.

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A More Extensions

A.1 New Customers Arriving in Period 2

Let $\gamma > 0$ be the share of customers that arrive in period 1. They stay for both periods. The share $1-\gamma$ arrives in period 2. For simplicity, assume that old and new customers are naive with probability α and sophisticated with $1-\alpha$. The whole analysis can easily be extended to new and old customers following different distributions. To make the extension interesting, assume that a firm n cannot distinguish new period-two customers from old ones that did not buy from n in period 1.[34] The results of this extension are summarized in the following Proposition:

Proposition 8. [New Customers in Period 2]
Shrouding equilibria exist. In each shrouding equilibrium that is Pareto dominant for the firms, consumers pay transparent prices $f_{n1} = c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\gamma\alpha(1-\alpha)\bar{a}$ *in the first period;* $f_{n2}^{soph} \geq c$ *and hidden prices are* $a_{n1} = a_{n2} = \bar{a}$*. If* $N \geq 3$*,* f_{n2}^{new} *and* f_{n2}^{naive} *are mixed on* $\left[c - \alpha\bar{a}, c - \frac{(1-\gamma)}{(1-\gamma)+(1-s_{max})\gamma(1-\alpha)}\alpha\bar{a}\right]$*. If* $N = 2$*,* $n \neq \hat{n}$ *choose prices such that* f_{n2}^{new} *and* $f_{\hat{n}2}^{naive}$ *are mixed on* $\left[c - \alpha\bar{a}, c - \frac{(1-\gamma)}{(1-\gamma)+(1-s_n)\gamma(1-\alpha)}\alpha\bar{a}\right]$*. Overall, firms earn profits* $\Pi_n = \frac{s_{min}}{1-s_{min}}s_n\gamma\alpha(1-\alpha)\bar{a} + s_n\gamma\alpha(1-\alpha)\bar{a}$*. Shrouding occurs with probability one.*

First of all, note that the nature of the support of the mixed prices changes from $N = 2$ to $N > 3$ due to a coordination problem: for $N = 2$, each firm has to make the other firm indifferent with her price choice. For $N > 3$, all $\hat{n} \neq n$ choose new-customer prices to make n indifferent in choosing naive-customer prices. This must be true for all n so that firms need to mix on the same supports. This support requires that no firm benefits from attracting the newly-arriving customers in equilibrium.

Comparing Proposition 3 and Proposition 8 shows that new customers in period 2 lower the upper bound of the interval on which new-customer and naive-customer prices are mixed in period 2. Overall, the profitability of old customers is unchanged, but they are fewer due to the normalization of the customer base. Competition for new customers is more fierce due to the newly arriving customers in the second period, driving down the upper bound of the interval on which new-customer prices (and therefore naive-customer prices) are mixed. But this leaves the profitability of old customers unaffected.

[34] If firms could make this distinction, markets for old and new costumers would be split. For the old customers, Corollary 1 would apply and the new ones would all get a transparent price of $c - \alpha\bar{a}$ in a shrouding equilibrium while only the naives pay hidden fees.

A.2 Learning about Hidden Fees in Period 2

To study the effects of learning by customers, assume that a share σ of naives remain naive while $1-\sigma$ become sophisticated in period 2.

Proposition 9. [Learning about hidden fees in Period 2]
Shrouding equilibria exist. In each shrouding equilibrium that is Pareto dominant for firms, consumers pay transparent prices $f_{n1} = c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\sigma\alpha(1-\sigma\alpha)\bar{a}$ *in the first period. Second period transparent prices are* $f_{n2}^{soph} \geq c$ *and* f_{n2}^{new} *and* f_{n2}^{naive} *are mixed on* $[c-\sigma\alpha\bar{a}, c]$*. Hidden prices are* $a_{n1} = a_{n2} = \bar{a}$*. Total profits are* $\Pi_n = \frac{s_{min}}{1-s_{min}} s_n\sigma\alpha(1-\alpha)\bar{a} + s_n\sigma\alpha(1-\alpha)\bar{a}$*. Shrouding occurs with probability one.*

Proposition 9 establishes that learning of some naive customers increases the second-period price floor and therefore the level of total prices for naive customers. The share of profitable naive customers decreases and firms, not knowing which first-period naive customer learns or remains naive, pay the naive-customer price to some old naives who are now sophisticated. This reduces the margin on old naives by the factor σ. Thus, there is cross-subsidization between the old naives. But as long as $\sigma > 0$, customer data are informative and profits remain positive.

Overall, effects on profits are the same as in the case with new customers arriving. Intuitively, customer data give a signal on which customers can be profitably exploited and which ones cannot. From this perspective, learning customers or new customers blur the informativeness of customer data in a similar way.

A.3 T Periods

Take $N = 2$ and denote by π_{nt} the profit in a shrouding equilibrium in period t of firm n. Similarly, denote by V_{nt} the continuation profit of such a firm when shrouding occurs in all forthcoming periods. Let $\underline{V}_t$ be the continuation profit of the firm with the smallest market share.

Proposition 10. [Deceptive Markets with Private Information about Customer Bases, $N = 2$ and $T > 2$ Periods]
A shrouding equilibrium with shrouding in each period exists. In this shrouding equilibrium, $f_{n1} = \min\{v, c - \alpha\bar{a} + \frac{\delta}{1-s_{min}}\underline{V}_t\}$ *and* $f_{nt}^{naive} = f_{nt}^{soph} = f_{\hat{n}t}^{new} = \min\{v, c - \alpha\bar{a} + \frac{\delta}{s_n} \cdot V_{\hat{n}t+1}\}$ *and* $\pi_{nt} = \min\{s_n(v + \alpha\bar{a} - c), \delta V_{\hat{n}t+1}\}$ *for all* $T > t > 1$*. Prices and profits in* T *are the same as in*

the Proposition 3.[35]

Profits remain positive on the shrouding equilibrium path if no firms learns about their competitor's customers, i.e. if no customer type switches. Thus, firms adjust prices for sophisticates and new-customers in order to prevent switching. These results might be quite stark, but they establish that profitable shrouding equilibria can be robust to models with more than two periods.

A.4 Mixed Strategies in Period 1

I emphasize in Proposition 4 that when hidden fees can be unshrouded, firms have an incentive to coordinate on prices in period 1. Proposition 7 establishes that this coordination incentive extends to the case when unshrouding of hidden fees is recognized by only some naives. For simplicity, I focus on pure-strategy equilibria in both cases, but mixed equilibria exist as well. In these equilibria, firms play the same finite number of first-period transparent prices with positive probability. In the case of Proposition 4, these prices must be within $\left[c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}\right]$ and with partial unshrouding within $\Big[c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}, c - \alpha$
Intuitively, if a firm would play a price with positive probability that no other firm sets with positive probability, shrouding does not occur when these prices realize and continuation profits are always reduced. By shifting probability mass from this price to another one which is played by all firms with positive probability, the firm can earn larger continuation profits and increase total expected profits. If firms would mix on an interval, coordination on the same price occurs with probability zero and large shrouding-continuation profits do not occur for these prices. But then, standard Bertrand arguments apply and drive prices downwards.

Since in each mixed-strategy equilibrium firms play the same finite number of transparent prices with positive probability, each price is played by all firms with positive probability as well. Thus, the coordination incentive to achieve large future shrouding profits prevails in mixed-strategy equilibria.

B Proofs

B.1 Proof of Proposition 1

Proof. I argue in the text that consumers buy at marginal cost in any pure-strategy equilibrium. The argument extends to mixed strategies here by a standard Bertrand argument as in the proof

[35] With last-period profits given, continuation profits and prices can be computed recursively.

for Lemma 1, Case (i). □

The proofs for Propositions 2 - 4 are done for the more general setup of Proposition 6 where unshrouding is more attractive. In addition to the basic framework, a share $\eta \in [0,1)$ of the naive consumers cannot avoid unshrouded hidden fees while the others can avoid them costlessly. The case $\eta = 1$ is ruled out to avoid that firms are indifferent between shrouding or not when only considering their own customer base. Thus, after unshrouding of hidden fees, the share $\alpha\eta$ of consumers still pays hidden fees while the share $1 - \alpha\eta$ does not. The special case presented in the text is obtained by setting $\eta = 0$.

B.2 Proof of Proposition 2

Since neither firm learns about consumers' types nor consumers about themselves, there is no updating of beliefs from any type; so the equilibrium is a SPNE. The relevant state variables are customer bases, represented by market shares in $t = 1$, and whether shrouding occurred in $t = 1$ or not.

Step 1: Period 2:

In the first step, I determine Nash equilibria of all period-2 subgames for all states.

Lemma 1 [Nash Equilibria in Period 2 Subgames]:

(i) After shrouding in period 1, a shrouding equilibrium exists if and only if

$$0 \geq \eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v - c\}. \tag{6}$$

Consumers pay hidden fees of $a_{n2} = \bar{a}$ and transparent prices $f_{n2} = c - \alpha\bar{a}$. Profits are zero. When $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v - c\} > 0$, hidden fees are unshrouded with probability one and consumers pay total prices equal to marginal costs.

(ii) After unshrouding in period 1, all consumer types pay total prices equal to marginal costs and hidden fees are zero.

Proof of Lemma 1. Case (i): In a first step, I derive the strategies of firms given all firms shroud hidden prices. In a second step, I derive conditions under which firms do not deviate from these strategies by unshrouding.

Given all firms shroud, two firms must set $f_{n2} = c - \alpha\bar{a}$ and $a_{n2} = \bar{a}$. Given all firms shroud, all firms with positive market share optimally set $a_{n2} = \bar{a}$ since this does not reduce demand but raises profits. I use a standard Bertrand-type argument to show that $f_{n2} = c - \alpha\bar{a}$ with probability one for at least two firms. One cannot have $f_{n2} \in (c - \alpha\bar{a}, \bar{f}_n]$ with positive probability for all firms for the supremum of transparent prices of firm n of $\bar{f}_n > c - \alpha\bar{a}$. Towards a contradiction, assume $\bar{f}_n > c - \alpha\bar{a}\ \forall n$. First note that $\bar{f}_n = \bar{f}\ \forall n$. Otherwise, a firm setting prices above the lowest supremum, say at $\bar{f}$, earns zero profits whenever these prices occur but could earn strictly positive profits by moving this probability mass to $\bar{f} - \epsilon$ for some $\epsilon > 0$ since $\bar{f} > c - \alpha\bar{a}$. Thus, if all firms have a supremum strictly above $c - \alpha\bar{a}$, they must have the same supremum. If all firms play $\bar{f}_n$ with positive probability, each firm earns non-negative profit when this occurs. But by taking the probability mass from $\bar{f}$ to $\bar{f} - \epsilon$, a firm could win the whole market when all others play $\bar{f}$ and therefore strictly increase her profit. If at least one firm does not play $\bar{f}$ with positive probability, all firms that do so earn zero profit with positive probability and could earn strictly positive profits by moving the probability mass somewhere below $\bar{f}$ instead. Therefore $f_{n2} < \bar{f}\ \forall n$ with probability one. But then profits go to zero as f_{n2} approaches $\bar{f}$ whereas expected profits are strictly positive by playing $c - \alpha\bar{a} + \epsilon$, for some $\epsilon > 0$, since all others play a larger price with positive probability when $\bar{f} > c - \alpha\bar{a}$. Thus, firms could do better by shifting probability mass from marginally below $\bar{f}$ to $c - \alpha\bar{a} + \epsilon$, for some $\epsilon > 0$. This is a contradiction. Hence, we get $\bar{f}_n = c - \alpha\bar{a}$ for at least two firms, since trivially, it is no equilibrium when only one firm sets $\bar{f}_n = c - \alpha\bar{a}$. Thus, firms earn zero profit when shrouding occurs.[36]

Given firms play a candidate shrouding equilibrium in which two firms set $\bar{f}_n = c - \alpha\bar{a}$ and $a_{n2} = \bar{a}$, unshrouding and setting $f_{n2} = c$ and $f_{n2} + a_{n2} = \min\{v, c + (1-\alpha)\bar{a}\}$ attracts all educated naives that cannot avoid hidden fees. Thus, optimal deviation profits by unshrouding are given by $\alpha\eta \cdot \min\{v - c, (1-\alpha)\bar{a}\}$. When $v - c > (1-\alpha)\bar{a}$ unshrouding is profitable if $\eta > 0$ and a shrouding equilibrium does not exist; if $\eta = 0$, optimal deviation profits by unshrouding are zero and a shrouding equilibrium exists. When $v - c < (1-\alpha)\bar{a}$, shrouding occurs as long as profits in a shrouding equilibrium are larger than profits from unshrouding, that is if $0 \geq \alpha\eta(v - c)$. If $v < c$, optimal deviation profits are negative and a shrouding equilibrium exists. Conversely, a shrouding equilibrium does not exist if $0 < \alpha\eta(v - c)$.

Next, I show that hidden fees are unshrouded with probability one when

[36] When I say below that a standard Bertrand type argument applies, I refer to this kind of reasoning.

$\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$ in three steps. Towards a contradiction, assume shrouding occurs with positive probability and $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$.

Step (I): *Firms earn positive profits.* When shrouding occurs, firms could unshroud and earn $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$, but since shrouding occurs with positive probability and firms must be indifferent between shrouding and unshrouding, firms must earn positive profits when shrouding occurs.

Step (II): *Firms earn zero profits whenever shrouding.* Let $\hat{t}$ be the supremum of total prices, including hidden fees when unshrouding, payed by educated naives that cannot avoid hidden fees. Then by playing $\hat{t}$, a firm earns positive profits only if it is the only one that unshrouds and $\hat{t} < f_{n2} + a_{n2}$ with positive probability. Thus, for all total prices above $\hat{t}$, firms earn positive profits only when shrouding occurs and they charge the smallest transparent price. But then, a standard Bertrand-type argument implies that total prices are competed downwards until $f_{n2} = c - \alpha\bar{a}$ for all firms that attract customers and $\hat{t} \leq \min\{(1-\alpha)\bar{a}, v-c\}$. Thus, firms earn weakly less than zero profits whenever shrouding.

Step (III): *Unshrouding occurs with probability one.* Since firms earn zero profits whenever shrouding, they are strictly better of by unshrouding instead since they can then earn $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$. Thus firms are better off by unshrouding with probability one, contradicting the assumption that shrouding occurs with positive probability whenever $\eta\alpha \cdot \min\{(1-\alpha)\bar{a}, v-c\} > 0$.

Note that the case depicted in Proposition 3 is for $\eta = 0$. Thus, unshrouding hidden prices can earn a firm maximally zero profits. Therefore, if $\eta = 0$ and shrouding occurs with positive probability, firms must earn zero profits when shrouding. If shrouding occurs with probability one, the result has been shown above. Suppose shrouding occurs with positive probability less then one. We know that unshrouding earns firms maximally zero profits. If at least one firm earns strictly positive profits when shrouding occurs, such a firm must have a supremum of transparent prices when shrouding of $\bar{f} > c - \alpha\bar{a}$. But then, a competitor could shift all probability mass from unshrouding to shrouding and earn strictly positive profits by setting a transparent price $\bar{f} - \epsilon$ for some $\epsilon > 0$ and hidden fees of $\bar{a}$. If all firms earn strictly positive profits when shrouding occurs, shrouding would occur with probability one since unshrouding gives zero profits. But then we are in the case from the beginning of this proof which contradicts positive profits. Thus, if $\eta = 0$ and shrouding occurs with positive probability, expected profits must be zero.

Case (ii): The market is effectively split: when unshrouding occurred in t=1, firms compete in transparent prices for sophisticated consumers and in total prices for unavoiding naives. By essentially the same Bertrand argument as above, firms that attract consumers charge $f_{n2} = c$ and $a_{n2} = 0$ and earn zero profits. $\square$

Step 2: Period 1:

All consumers face the same price-schedule in period 2, irrespective of the firm they purchase from. Thus, consumers maximize their total payoff by maximizing their first-period payoff. Knowing that firms earn no profits in any second-period subgames, firms simply maximize their per-period profit in period 1. Thus, the same Bertrand-type argument as in Case (i) of period 2 applies.

B.3 Proof of Proposition 3

I am looking for a Perfect Bayesian Equilibrium. In the first step, I argue that updating of beliefs only matters for the firms' customer base after shrouding in period 1. After such histories, firms learn only their own first period customers' types. Thereafter, I determine conditions for shrouding to occur in equilibrium in period 2 and pin down consumers' payments and firms' profits. Those are summarized in Lemma 3.

Step 1: After shrouding occurred in period 1, firms update only about consumers in their customer base.

Assume shrouding occurred in period 1. When consumers are not educated about hidden fees, both consumer types solve the same problem: $\max_{\mathrm{n}} v - f_{n2}, s.t. v - f_{n2} \geq 0$. Hence, both consumer types will always be indifferent between the same set of firms. Therefore the Sorting Assumption implies that the distribution of customers in each customer base is the same as in the population. Hence from observing her own customer base, a firm cannot learn anything about the distribution outside of her own customer base.

Recall that after unshrouding in period 1, all consumers are sophisticated in period 2, and this is known to firms.

Step 2: Period 2

To derive second-period equilibria, I begin by establishing some characteristics of the firms' second-period pricing distributions.

Lemma 2 [Supports of Transparent Prices in Period 2]: In each equilibrium in which prices remain shrouded in period 2 with probability one, $f_{n2}^{naive} \in [c - \alpha\bar{a}, c]$ with probability one and sophisticates pay a price below c, i.e. $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c \; \forall n$ with probability one. $F_n^{new}(.)$ and $F_n^{naive}(.)$ are continuous on $(c - \alpha\bar{a}, c)$, and on each subinterval on $(c - \alpha\bar{a}, c)$ at least one firm plays naive- and one firm plays new-customer prices with positive probability. Additionally, all firms play marginally undercut c with the new-customer price with positive probability, i.e. for all $\epsilon > 0$ and for all n, $f_{n2}^{new} \in (c-\epsilon, c]$ with positive probability. Furthermore, $F_n^{new}(c - \alpha\bar{a}) = F_n^{new}(c - \alpha\bar{a}) = 0, \; \forall n$.

Proof of Lemma 2.

f_{n2}^{naive} is in $[c - \alpha\bar{a}, c]$ and $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c \; \forall n$ with probability one. I have argued in the main body that in each equilibrium in which prices remain shrouded in the second period, $f_{n2}^{soph} \geq c$, $f_{n2}^{new} \geq c - \alpha\bar{a}$ and $f_{n2}^{naive} \geq c - \alpha\bar{a}$. First, I show that in equilibrium no firm n sets a price $f_{n2}^{naive} > c$ with positive probability. A firm n can guarantee itself strictly positive expected profits from its naive customers by setting $c - \alpha\bar{a}$. Thus, it must earn strictly positive expected profits for almost all prices it charges, and any price it charges with positive probability. Let $\bar{f}_{n2}^{naive}$ be the supremum of those prices and suppose $\bar{f}_{n2}^{naive} > c$ with positive probability. Then, all rivals $\hat{n} \neq n$ must set prices $f_{\hat{n}2}^{new} \geq \bar{f}_{n2}^{naive}$ with positive probability. If all rivals do so, each firm $\hat{n} \neq n$ can deviate and move probability mass from weakly above $\bar{f}_{n2}^{naive}$ to $\bar{f}_{n2}^{naive} - \epsilon$, and for sufficiently small ϵ increase its profits. We conclude that $f_{n2}^{naive} \leq c \; \forall n$.

To show that $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c \; \forall n$, I first establish that firms earn zero expected profits from new-customers. Towards a contradiction, suppose a firm n makes positive expected profits from new customers and take its supremum of new-customer prices $\bar{f}_{n2}^{new}$. To be profitable at $\bar{f}_{n2}^{new}$, $\bar{f}_{n2}^{new} > c - \alpha\bar{a}$. In addition, there must be a firm $\hat{n} \neq n$ such that $f_{\hat{n}2}^{soph} > \bar{f}_{n2}^{new}$ or $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with positive probability. If $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with positive probability, $\hat{n}$ gets zero profits from naives whenever playing $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$. By moving this probability mass to $\bar{f}_{n2}^{new} - \epsilon$ for sufficiently small $\epsilon > 0$ instead, $\hat{n}$ could make strictly positive profits, a contradiction. The same argument applies if $f_{\hat{n}2}^{soph} > \bar{f}_{n2}^{new}$ with positive probability. Hence, new-customer prices earn zero expected profits in equilibrium. This directly implies that firms earn zero profits on their old sophisticates as well: otherwise, by the same reasoning as above, a firm could move the probability mass of its new-customer prices from above the supremum of sophisticates' prices of the positive-profit firm to minimally below it, and thereby increase its profits. It follows that $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c \; \forall n$ with probability one. If $f_{n2}^{soph} > c$ with positive probability, then at least one firm $\hat{n} \neq n$ must set $f_{\hat{n}2}^{new} \leq c$ with probability one, since otherwise a competitor of n would get strictly positive expected profits from new-customer prices. Similarly, if all $(f_{\hat{n}2}^{new})_{\hat{n}\neq n} > c$ with positive probability, then $f_{n2}^{soph} \leq c$ with probability one for $\hat{n}$ not to earn strictly positive profits with new-customer prices. Hence, $min\{f_{n2}^{soph}, (f_{\hat{n}2}^{new})_{\hat{n}\neq n}\} \leq c$ for all n, and we established that the support of f_{n2}^{naive} is $[c - \alpha\bar{a}, c]$.

On each subinterval on $(c-\alpha\bar{a}, c)$, at least one firms plays naive-, and at least one firm plays new-customer prices with positive probability. All firms play new-customer prices arbitrarily close to c with positive probability. I prove the claim in three steps: first, I establish that in any arbitrarily small interval $(c - \epsilon, c]$ at least two firms play naive- and all firms play new-customer prices with positive probability. Second, I show the same for any arbitrarily small interval $[c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ for at least two firms' naive- and two firms' new-customer prices. Third, I prove that on each interval in-between these prices occur with positive probability.

Step (i): First, I show that for all n and any $\epsilon > 0$, $f_{n2}^{new} \in (c-\epsilon, c]$ with positive probability. Suppose otherwise, i.e. for at least one firm there exists an $\epsilon > 0$ such that $f_{n2}^{new} \in (c-\epsilon, c]$ with probability zero. Of all of these firms, select a firm n that has the smallest supremum $\bar{f}_{n2}^{new}$. If there are many such firms select one that sets the supremum with probability less than one. Since $\bar{f}_{n2}^{new} < c$, at least one firm $\hat{n} \neq n$ must set $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with positive probability for n to break even. But then, $\hat{n}$ makes zero profit for all $f_{\hat{n}2}^{naive} > \bar{f}_{n2}^{new}$ with probability one, a contradiction. Thus, for any $\epsilon > 0$ all firms set $f_{n2}^{new} \in (c - \epsilon, c]$ with positive probability. It follows that for every $\epsilon > 0$ and every n, some $\hat{n} \neq n$ sets $f_{n2}^{naive} \in (c - \epsilon, c]$ with positive probability: otherwise, firms could not break even when setting $f_{n2}^{new} \in (c - \epsilon, c)$ with positive probability. Since this holds for every n and $\epsilon > 0$, at least two firms set naive-customer prices in any interval $(c-\epsilon, c]$. Thus, for all prices in $(c - \alpha\bar{a}, c)$, every firm sets larger new-customer with positive probability, and at least two firms set larger naive-customer prices with positive probability.

Step (ii): First I show that for every $\epsilon > 0$, at least two firms set $f_{n2}^{naive} \in [c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ with positive probability. Suppose otherwise and take a firm n and her competitors $\hat{n} \neq n$. Assume towards a contradiction that there exists an $\epsilon > 0$ such that for all $\hat{n}$, $f_{\hat{n}2}^{naive} \in [c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ with probability zero. Then the infimum of the naive-customer prices of n's competitors $\underline{f}$ satisfies $\underline{f} > c - \alpha\bar{a}$. For naive-customer prices above this infimum to be profitable, all new-customer prices must be larger with positive probability. But then firm n can earn strictly positive profits from new-customers by choosing $f_{n2}^{new} \in (c - \alpha\bar{a}, \underline{f})$ with probability one. But this contradicts the finding that firms earn zero expected profits from new-customers. Since this is true for all

n, I conclude that for every $\epsilon > 0$, at least two firms set $f_{n2}^{naive} \in [c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ with positive probability. To show that the same is true for new-customer prices, suppose towards a contradiction that there exists an $\epsilon > 0$ such that a firm n plays $f_{n2}^{naive} \in [c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ with positive probability but all $\hat{n} \neq n$ play greater new-customer prices with probability one. But then, n could move its probability mass from below $c - \alpha\bar{a} + \epsilon$ onto this point to strictly increase profits. Thus, we get a contradiction if for any $\epsilon > 0$, less then two firms play $f_{n2}^{new} \in (c - \alpha\bar{a}, c - \alpha\bar{a} + \epsilon)$ with positive probability.

Step (iii): On each subinterval on $(c - \alpha\bar{a}, c)$, at least one firm sets naive- and at least one other firm sets new-customer prices with positive probability. Suppose the opposite for some interval $(\widetilde{r}, \widetilde{s})$. Then there are three cases: either no naive- and new-customer price on $(\widetilde{r}, \widetilde{s})$ occurs with positive probability, or only naive-customer prices, or only new-customer prices. Take the largest interval containing $(\widetilde{r}, \widetilde{s})$, in which either no firm sets new- or no firms sets naive-customer prices with positive probability, and denote it by (r, s); i.e., some new- or naive-customer prices are played with positive probability arbitrarily close below r and arbitrarily close above s. Note that due to step (ii), we know that $r > c - \alpha\bar{a}$.

In the first case, no naive- or new-customer price occurs on (r, s) with probability. But by construction, some naive- or new-customer price occurs on $(r - \epsilon, r]$ with positive probability. Note that there can be no mass point on r. If more than one firm had amass-point on r, they could strictly increase profits by shifting probability mass from this mass point to slightly below it. If one firm had a mass point on r, it could shift this mass point upwards into (r, s) and increase margins without affecting expected market shares since (r, s) is empty. But when there is no mass point on r, then for some $\epsilon > 0$ small enough, a firm playing prices in $(r - \epsilon, r]$ with positive probability is strictly better off by shifting this probability mass to slightly below s, a contradiction.

Now consider the second case. Towards contradiction, assume only naive-customer prices are set on (r, s) with positive probability. But by shifting probability mass of naive-customer price from within (r, s) to s, firms can discretely increase margins on naives while leaving the probability to gain these margins unaffected, a contradiction.

Third, assume towards a contradiction that only new-customer prices are played on (r, s) with positive probability. If only one firm plays new-prices on (r, s) with positive probability, this firm could strictly increase its profits by moving this probability mass to slightly below s, a contradiction. Now suppose at least two firms play new-customer prices on (r, s) with positive probability. Take a firm n playing price $f \in (r,s)$ and $f' \in (r,s)$ with positive probability where $f \neq f'$. Recall that both prices are the smallest new-customer price with positive probability due to Step (i), and earn zero expected margins in this case, as shown in the beginning of this proof. Since no naive-customer prices occurs with positive probability on (r,s), both prices induce exactly the same probability of attracting naives when being the smallest new-customer price. But since one of these prices is strictly larger, they cannot both have zero expected margins when being the smallest new-customer price, a contradiction.

The CDFs are continuous in the interior of the support, i.e. F_n^{new} and F_n^{naive} have no mass point on $(c-\alpha\bar{a}, c)$, $\forall n$. Take F_n^{new} and suppose otherwise. Pick the lowest mass-point of all firms. Say n has this mass point at f. We know from above that larger naive-customer prices occur with positive probability, so that prices at this mass point are payed with positive probability. Then there exists some $\epsilon > 0$ such that no rival $\hat{n} \neq n$ charges a price $f_{\hat{n}2}^{naive}$ in $[f, f+\epsilon)$. For otherwise, a firm $\hat{n}$ that sets $f_{\hat{n}2}^{naive} \in [f, f+\epsilon)$ could charge $f-\epsilon$ instead; as $\epsilon \to 0$, the price difference goes to zero but $\hat{n}$ wins with higher probability. But when no rival charges a naive-customer price in $[f, f+\epsilon)$ and only n sets a mass-point of new-customer prices at f, then n can increase profits by moving the mass point upwards, a contradiction. Alternatively, another firm but n has a mass point on new-customer prices at f as well. Recall that profits from new-customers are zero in expectation. Thus, by shifting the mass point upwards, n looses more often, gaining zero profits in this case; but due to Step (i), n still has the lowest new-customer prices with positive probability and therefore earns a strictly positive margin when attracting customers, a contradiction. This shows that F_n^{new} has no mass point on $(c-\alpha\bar{a}, c)$. A similar argument applies to F_n^{naive}: to see why, suppose otherwise that F_n^{naive} has a mass point on $(c-\alpha\bar{a}, c)$. Pick again the lowest mass point of all firms. Say firm n has this mass point at f. By the same argument as above, there exists some $\epsilon > 0$ such that no rival $\hat{n} \neq n$ sets a price $f_{\hat{n}2}^{new} \in [f, f+\epsilon)$ with positive probability. And since n only competes with these new-customer prices for its naive customers, n can strictly improve profits by shifting the mass point upwards, a contradiction.

One has $F_n^{new}(c-\alpha\bar{a}) = F_n^{new}(c-\alpha\bar{a}) = 0$, $\forall n$. Suppose otherwise, i.e. $F_n^{new}(c-\alpha\bar{a}) = p > 0$ for some n. Then no rival $\hat{n} \neq n$ charges $f_{\hat{n}2}^{naive} \in (c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ for some $\epsilon > 0$, or otherwise $\hat{n}$ could strictly increase profits by moving this probability-mass on $c-\alpha\bar{a}$ instead. But then, by the same argument as in the last paragraph, n can earn strictly positive profits by shifting the mass-point upwards, a contradiction.

Now suppose $F_n^{naive}(c-\alpha\bar{a}) = p > 0$ for some n and take firms $\hat{n} \neq n$ that play new-customer prices on $(c-\alpha\bar{a}, c-\alpha\bar{a}+\epsilon)$ with positive probability. We already know that such firms exist. Then $\hat{n}$'s profits from $f_{\hat{n}2}^{new} = c - \alpha\bar{a} + \epsilon$ converge to some profit-level below $p[s_n(1-\alpha)(c - \alpha\bar{a} - c) + (1-s_n)0] + (1-p)0 = -ps_n\alpha\bar{a} < 0$. This is a contradiction since firms can guarantee themselves at least zero profits from new-customer prices.

□

The next lemma summarizes the properties in each shrouding equilibrium in period 2 for each state.

Lemma 3 [Second Period Continuation Equilibria]
There always exists the standard Bertrand equilibrium in which at least two firms unshroud and each consumers pays marginal costs. In addition to this equilibrium, there exist second-period continuation equilibria in which shrouding occurs with positive probability under the following conditions:

(i) If shrouding occurred in t=1 and all firms have positive customer bases, shrouding occurs with positive probability if and only if

$$s_n\alpha(1-\alpha)\bar{a} \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v - c\}, \ \forall n. \tag{7}$$

In such a shrouding equilibrium, profits are $s_n\alpha(1-\alpha)\bar{a}$ and shrouding occurs with probability one. f_{n2}^{naive} is mixed as in (2). Switching naive-customers of firm n's customer base pay the smallest new-customer prices of n's competitors based on (1). Sophisticated customers in the customer base of firm n pay a price equal to the smallest new-customer price of n's competitors based on (1). When the above shrouding condition is violated, unshrouding occurs with probability one and all consumers pay a price of c.

(ii) If shrouding occurred in t=1 and some firm has an empty customer base, consumers are educated about hidden fees with probability one if and only if the good is socially desirable and $\eta > 0$. In this case, prices equal marginal costs and firms make zero profits. If the product is socially wasteful, prices are as in (i), but firms without customer base make zero profits. If $\eta = 0$, firms without customer base are indifferent between shrouding or unshrouding.

Proof of Lemma 3.

(i) First, I derive shrouding conditions and pin down the level of equilibrium profits in a shrouding equilibrium in which firms have a positive customer base. Then, I construct the mixed equilibrium strategies for period 2 in the shrouding equilibrium based on (1) and (2).

If $s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ $\forall n$, in all equilibria in which shrouding occurs with positive probability it occurs with probability one. If $s_n\alpha(1-\alpha)\bar{a} < \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ for some n, shrouding occurs with probability zero. If shrouding occurs with probability one, firms earn expected profits of $s_n\alpha(1-\alpha)\bar{a}$ from naives and zero from sophisticates and new customers. Suppose that shrouding occurs with positive probability. I show that this implies Step (I) - (III) below. Using these facts Step (IV) proves the above.

Step (I): *Firms earn positive profits.* When shrouding occurs, firms can get positive profits of at least $s_n\alpha(1-\alpha)\bar{a}$ by setting $f_{n2}^{soph} = f_{n2}^{new} = c$ and $f_{n2}^{naive} = c - \alpha\bar{a}$. I have established in the text that when shrouding occurs, new-customer prices below $c - \alpha\bar{a}$ are never played as they lead to strictly negative profits for at least one firm. When unshrouding, the share of consumers paying a hidden fee reduces to $\eta\alpha$ and this threshold shifts upwards to $c - \eta\alpha\bar{a}$. Thus, firms can indeed be sure to profitably keep its naive customers when shrouding occurs by setting the above prices. Since shrouding occurs with positive probability, firms make positive expected profits.

Step (II): *New-customer prices earn zero expected margins in equilibrium conditional on both shrouding or unshrouding occurring. Sophisticated consumers never pay positive margins in equilibrium.* Towards a contradiction, suppose a firm n profitably attracts customers with her new-customer price in expectation. Then firm n must earn positive expected margins with each new-customer price that is played with positive probability. Take the supremum of these prices $\bar{f}_{n2}^{new}$. Then prices that minimally undercut $\bar{f}_{n2}^{new}$, i.e. prices on $(\bar{f}_{n2}^{new} - \epsilon, \bar{f}_{n2}^{new}]$ for some sufficiently small $epsilon > 0$, profitably attract either sophisticates or naives from another firm, say $\hat{n} \neq n$. We therefore have to distinguish these two cases.

Suppose n profitably attracts sophisticates conditional on shrouding in any interval of new-customer prices that marginally undercut $\bar{f}_{n2}^{new}$. Then $f_{\hat{n}2}^{soph} \geq \bar{f}_{n2}^{new}$ with positive probability. Note that the inequality must be strict for some $f_{\hat{n}2}^{soph}$ when n sets $\bar{f}_{n2}^{new}$ with positive probability. Then $\hat{n}$ earns zero profits from sophisticates with probability one whenever $f_{\hat{n}2}^{soph} \geq \bar{f}_{n2}^{new}$, though $\hat{n}$ could earn strictly positive profits from sophisticates when shifting this probability mass to $\bar{f}_{n2}^{new} - \epsilon$ for some small enough $\epsilon > 0$, a contradiction. The exact same argument applies conditional on unshrouding occurring.

Now suppose n profitably attracts naives in any interval of new-customer prices arbitrarily close below $\bar{f}^{new}_{n2}$. They are profitable when shrouding occurs or when unshrouding occurs so that I have to distinguish these two cases. If they are profitably attracted under shrouding, we must have $f^{naive}_{\hat{n}2} \geq \bar{f}^{new}_{n2}$ with positive probability. Note that the inequality must be strict for some $f^{naive}_{\hat{n}2}$ when $\bar{f}^{new}_{n2}$ occurs with positive probability. Then $\hat{n}$ earns zero profits when shrouding occurs on prices $f^{naive}_{\hat{n}2} \geq \bar{f}^{new}_{n2}$ that occur with positive probability. W.l.o.g. let $\bar{f}^{new}_{n2}$ be among the largest such suprema. If this was not the case, then another firm would have a larger supremum that earns zero profits for prices that marginally undercut it. But then this firm could do strictly better by shifting this probability mass to $\bar{f}^{new}_{n2}$. Thus $\bar{f}^{new}_{n2}$ can be taken among the largest suprema w.l.o.g.. But then moving probability mass from $[\bar{f}^{new}_{n2}, \bar{f}^{new}_{n2} + \epsilon)$ to $\bar{f}^{new}_{n2} - \epsilon$ increases $\hat{n}$'s profits discretely when shrouding occurs and reduces them by maximally 2ϵ when unshrouding occurs. This is profitable for some small enough $\epsilon > 0$, a contradiction. If n profitably attracts naives when unshrouding occurs, the same argument can be applied to total prices, i.e. by taking $t^{naive}_{\hat{n}2} = f^{naive}_{\hat{n}2} + a_{\hat{n}2}$ and $t^{new}_{n2} = f^{new}_{n2} + a_{n2}$ with $\bar{t}^{new}_{n2}$ as the supremum to total new-customer prices of firm n.

I conclude that if shrouding occurs with positive probability, new-customer prices earn zero expected profits conditional on shrouding or unshrouding. To show that sophisticated consumers never pay a price $f^{soph}_{n2} > c$, suppose otherwise. Since I have established that sophisticates never pay a new-customer price $f^{new}_{\hat{n}2} > c$, they must pay the positive margin to their old firm, i.e. with $f^{soph}_{n2} > c$. But then, a competitor can earn strictly positive profits with new-customer prices by offering $f^{new}_{\hat{n}2} = f^{soph}_{n2} - \epsilon$ for some $\epsilon > 0$ small enough, a contradiction.

Step (III): *The profits of firms that shroud are weakly smaller than* $s_n\alpha(1-\alpha)\bar{a}$ $\forall n$ *and zero when unshrouding occurs.* To show that firms' profits are weakly smaller than $s_n\alpha(1-\alpha)\bar{a}$ when shrouding occurs, suppose otherwise, i.e. there exists a firm n that earns strictly larger profits when shrouding occurs. Step (II) has established that firms earn zero profits from new- and sophisticated customers, therefore positive profits have to be earned from naive customers from a firm's customer base. Let $\bar{f}^{naive}_{n2}$ be the supremum of n's naive-customer prices that are payed with positive probability. Then all $\tilde{n} \neq n$ must set $f^{new}_{\tilde{n}2} \geq \bar{f}^{naive}_{n2}$ with positive probability. I.e. for all $\epsilon > 0$, some $\hat{n} \neq n$ sets $f^{new}_{\hat{n}2} \in [\bar{f}^{naive}_{n2}, \bar{f}^{naive}_{n2} + \epsilon)$ with positive probability. But by moving probability mass from this interval to $\bar{f}^{naive}_{n2} - \epsilon$, $\hat{n}$ can make strictly positive profits: if some other firm than $\hat{n}$ sets a smaller new-customer price, $\hat{n}$ earns zero profits from new customers. But since all $\tilde{n} \neq n$, $\tilde{n} \neq \hat{n}$ set $f^{new}_{\tilde{n}2} \geq \bar{f}^{naive}_{n2}$ with positive probability, $f^{new}_{\hat{n}2} = \bar{f}^{naive}_{n2} - \epsilon$ is the smallest new customer price with positive probability. In this case, $\hat{n}$ earns profits strictly above $s_n\alpha(1-\alpha)\bar{a}$

in expectation from n's naives and looses weakly below $s_n\alpha(1-\alpha)\bar{a}$ from n's sophisticates. Note that we know from Step (I) that $f^{new}_{\hat{n}2} \geq c - \alpha\bar{a}$ and therefore $f^{naive}_{\hat{n}2} \geq c - \alpha\bar{a}$ for all $\hat{n}$, which is why losses from attracting sophisticates from firm n are weakly below $s_n\alpha(1-\alpha)\bar{a}$. From all other sophisticates that $\hat{n}$ attracts with this price, it looses maximally 2ϵ. Thus, for some $\epsilon > 0$ small enough, $\hat{n}$ can discretely increase profits by shifting some probability mass from $f^{new}_{\hat{n}2} \in [\bar{f}^{naive}_{n2}, \bar{f}^{naive}_{n2} + \epsilon)$ to $\bar{f}^{naive}_{n2} - \epsilon$, a contradiction.

To show that shrouding firms earn zero profits conditional on unshrouding, suppose otherwise for at least one firm, say n. Step (II) implies that these profits must be earned from naive customers of firm n's customer base. Thus, n must keep some unavoiding naives at a positive total prices $f^{naive}_{n2} + a_{n2} > c$. But then, a competitor $\hat{n} \neq n$ can earn strictly positive profits from new-customer prices conditional by unshrouding and setting $f^{new}_{\hat{n}2} + a_{\hat{n}2} = c + \epsilon$ for some sufficiently small $\epsilon > 0$, which contradicts Step (II). Thus, shrouding firms earn zero profits conditional on unshrouding. Since firms' profits are weakly below $s_n\alpha(1-\alpha)\bar{a}$ when shrouding but by Step (I) they can guarantee themselves these profits when shrouding occurs, we know that firms must earn profits of $s_n\alpha(1-\alpha)\bar{a}$ in expectation when shrouding occurs.

Step (IV): *If $s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ $\forall n$, in all equilibria in which shrouding occurs with positive probability, it occurs with probability one. If $s_n\alpha(1-\alpha)\bar{a} < \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ for at least one n, shrouding occurs with probability zero.* Steps (I)-(III) establish that expected profits from new customers are zero, whether shrouding or unshrouding occurs, and whenever shrouding, firms' expected profits are $s_n\alpha(1-\alpha)\bar{a}$ when shrouding occurs and zero when unshrouding occurs. Thus, in any candidate equilibrium in which shrouding occurs with positive probability, it occurs with probability one. Consequently, when $s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$ $\forall n$, no firm has an incentive to unshroud with probability one and set a total price of $\min\{c + (1-\alpha)\bar{a}, v\}$. But when this condition is violated for at least one firm, this firm has a strict incentive to unshroud with probability one and set the above total price.

Now that I established that in any second-period continuation equilibrium where shrouding occurs, it occurs with probability one, I can use the properties on new- and naive-customer distributions derived in Lemma 2 and the profit levels pinned down above to construct equilibrium price-distributions.

Mixed strategies for new-customer prices. Recall that firms do not compete for their own old customers with the new-customer price. When a firm n sets her naive-customer price lower than all her competitors' new-customer prices, it keeps her naive customers. Otherwise, it looses them. Thus, expected profits are

$$(1-\prod_{j\neq n}(1-F_j^{new}(f_{n2}^{naive})))\cdot 0+\prod_{j\neq n}(1-F_j^{new}(f_{n2}^{naive}))\cdot s_n\alpha(f_{n2}^{naive}+\bar{a}-c)=const.\quad ,\forall n. \tag{8}$$

We know from Lemma 2 that all new- and naive-customer prices on $(c-\alpha\bar{a},c)$ occur with positive probability and that $F_j^{new}(c-\alpha\bar{a})=0$ for all j. We also know that expected profits from naive-customer prices must be equal to $const.=s_n\alpha(1-\alpha)\bar{a}$ for all prices on the interval. Thus, I can rewrite the above to get

$$\prod_{j\neq n}(1-F_j^{new}(f_{n2}^{naive}))=\frac{(1-\alpha)\bar{a}}{f_{n2}^{naive}+\bar{a}-c}\quad ,\forall n \tag{9}$$

In particular, for each $\hat{n}\neq n$ and f_{n2}^{naive} this requires $\prod_{j\neq\hat{n}}(1-F_j^{new}(f^{naive}))=\prod_{j\neq n}(1-F_j^{new}(f^{naive}))$, which implies $F_n^{new}(f^{naive})=F_{\hat{n}}^{new}(f^{naive})=F^{new}(f^{naive})$. Using this symmetry in the above equation leads to the expression of (1) on $(c-\alpha\bar{a},c)$.

Note that the probability mass below c is not equal to one. In fact, we only know from Lemma 2 that $min\{f_{n2}^{soph},(f_{\hat{n}2}^{new})_{\hat{n}\neq n}\}\leq c\ \forall n$ with probability one. New-customer prices can be strictly larger than c with positive probability, but these prices are never payed by customers and are therefore inconsequential for consumer welfare and firms' profits. Thus, either new-customer prices have a mass point at c and sophisticated customer prices can be strictly larger than c or the other way around. I report the strategy with the mass point on c to ease the exposition of results. This leads to the distribution as in (1).

Mixed strategies for naive-customer prices. Take a firm n that sets f_{n2}^{new} to all consumers that are not in n's customer base. In order to win firm j's customers and break even, it has to offer a new-customer price f_{n2}^{new} such that (i) $f_{n2}^{new}<f_{\hat{n}2}^{new}\forall\hat{n}\neq j$ and (ii) $f_{n2}^{new}<f_{j2}^{naive}$. If f_{n2}^{new} is such that (i) is satisfied, but j's naive-customer price is still smaller, than n attracts only the sophisticated consumers of j, since $f_{j2}^{soph}\geq c$. Hence, the expected profit of attracting j's customers is

$$\begin{aligned}(1-F^{new}(f_{n2}^{new}))^{N-2}[(1-F_j^{naive}(f_{n2}^{new}))s_j(f_{n2}^{new}+\alpha\bar{a}-c)\\+F_j^{naive}(f_{n2}^{new})s_j(1-\alpha)(f_{n2}^{new}-c)]\end{aligned} \tag{10}$$

Summing over all $j \neq n$ leads to n's expected profits from new-customer prices:

$$(1-F^{new}(f_{n2}^{new}))^{N-2}[(f_{n2}^{new}+\alpha\bar{a}-c)\sum_{j\neq n}(1-F_j^{naive}(f_{n2}^{new}))s_j \\ +(1-\alpha)(f_{n2}^{new}-c)\sum_{j\neq n}F_j^{naive}(f_{n2}^{new})s_j] = const. \tag{11}$$

Lemma 2 established that all naive-customer prices on $(c-\alpha\bar{a},c)$ occur with positive probability and that $F^{new}(c-\alpha\bar{a}) = F_j^{new}(c-\alpha\bar{a}) = 0$. I have shown above that expected profits from new-customer prices are zero. Now consider $f_{n2}^{naive} \in (c-\alpha\bar{a},c)$. Rewriting the equation gives

$$\sum_{j\neq n}F_j^{naive}(f_{n2}^{new})s_j = (1-s_n)\frac{(f_{n2}^{new}+\alpha\bar{a}-c)}{\alpha(f_{n2}^{new}+\bar{a}-c)}, \quad \forall n \tag{12}$$

$$\Leftrightarrow \underbrace{\sum_{j=1}^{N}F_j^{naive}(f_{n2}^{new})s_j}_{\equiv g(f)} = (1-s_n)\underbrace{\frac{(f_{n2}^{new}+\alpha\bar{a}-c)}{\alpha(f_{n2}^{new}+\bar{a}-c)}}_{\equiv\Omega(f_{n2}^{new})} + s_nF_n^{naive}(f_{n2}^{new}), \quad \forall n \tag{13}$$

$$\Leftrightarrow g(f_{n2}^{new}) = (1-s_n)\Omega(f_{n2}^{new}) + s_nF_n^{naive}(f_{n2}^{new}), \quad \forall n \tag{14}$$

For each n, the condition implies $F_n^{naive}(f_{n2}^{new}) = \frac{g(f)}{s_n} - \frac{1-s_n}{s_n}\Omega(f)$. Plugging this into (11) pins down $g(f) = \Omega(f)$ for all f and therefore $F_n^{naive}(f_{n2}^{new}) = \Omega(f)$. Hence, in all second-period shrouding equilibria, naive customer prices are mixed symmetrically according to (2).

(ii) I show in this section that after histories in which shrouding occurs and at least one firm has no customer base and another has one, firms always educate about hidden fees if the product is socially desirable and $\eta > 0$. Firms make no profit and consumers pay marginal costs.

Given shrouding occurs with positive probability, the same reasoning as in (i) implies that firms can earn $\widetilde{s}_n\alpha(1-\alpha)\bar{a}$ conditional on shrouding from their old naive customers while firms earn zero expected profits from new-customer prices and old sophisticates.[37] But then firms without a customer base earn zero total profit since they have no customer base to exploit and their shrouding condition reduces to $0 \geq \eta\alpha\min\{(1-\alpha)\bar{a}, v-c\}$. As long as $v > c$ and $\eta > 0$, they have a strict incentive to educate customers about hidden fees. When η is equal to zero, profits are zero after unshrouding. Firms without customer base are indifferent between shrouding and unshrouding and there are potentially multiple equilibria.

[37] $\widetilde{s}_n(\geq s_n)$ is the market share a firm gets when not all firms sell to consumers but n does.

(iii) After all other histories, hidden fees are unshrouded in period 1. Thus, standard Bertrand arguments imply that each consumer pays marginal costs.

□

B.4 Proof of Proposition 4

The results of Proposition 3 pin down the continuation payoffs after period 1 and can be used to study equilibrium behavior in period 1.

Lemma 3 establishes that when $v > c$ and $\eta > 0$, firms can achieve positive continuation profits if and only if each firm has a positive customer base, i.e. when prices in the first period are identical with positive probability.

First, I study equilibria in which firms always set the same transparent price f_1 in the first period. Given the reduced-game profits starting from $t = 1$ specified in (3), the only possible profitable deviations are either (i) shrouding and undercutting competitors or (ii) unshrouding hidden fees and attracting the remaining profitable customers.

(i) is unprofitable if $s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq f_1 + \alpha\bar{a} - c$, which is equivalent to $f_{n1} \leq c - \alpha\bar{a} + \frac{s_n}{1-s_n}\alpha(1-\alpha)\bar{a}$.

To check for (ii), I need to establish the optimal deviation under unshrouding. Given all other firms shroud and play f_1, a firm n can make sure to attract only all profitable customers after unshrouding, i.e. only all unavoiding naives and neither educated avoiding naives nor sophisticates, by setting $\widetilde{f_1} > \max\{c, f_1\}$ and $\widetilde{a_1} < \min\{f_1 + \bar{a}, v\} - \widetilde{f_1}$. The resulting deviation profits are bounded by $\eta\alpha\min\{f_1 + \bar{a} - c, v - c\}$. Note that I do not have to consider the case where $f_1 \geq c$ since the resulting deviation profits of $\eta\alpha(f_1 + \bar{a} - c) \leq f_1 + \alpha\bar{a} - c$ for all $f_1 \geq c$, and therefore deviation (i) is always preferred. Thus, consider $f_1 < c$, in which case only unavoiding naives are profitable after unshrouding. Hence, the optimal deviation profits with unshrouding are $\eta\alpha\min\{f_1 + \bar{a} - c, v - c\}$. Deviating in this way is unprofitable if $s_n(f_1 + \alpha\bar{a} - c) + s_n\alpha(1-\alpha)\bar{a} \geq \eta\alpha\min\{f_1 + \bar{a} - c, v - c\}\ \forall n$. Thus I have to consider three cases. First, if $f_1 + \bar{a} < v$ and $s_n < \eta\alpha\ \forall n$, we get $f_1 \leq c - \alpha\bar{a} + \frac{\eta\alpha + s_n(1-\alpha)}{\eta\alpha - s_n}\alpha\bar{a}\ \forall n$. This is always larger than $c - \alpha\bar{a} + \frac{s_n}{1-s_n}\alpha(1-\alpha)\bar{a}$, the upper bound from (ii), which is why (i) does not need to be considered in this case. Second, if $f_1 + \bar{a} < v$ and $s_{max} > \eta\alpha$, we get a lower bound for prices of $f_1 \geq c - \alpha\bar{a} - \frac{\eta\alpha + s_n(1-\alpha)}{s_n - \eta\alpha}\alpha\bar{a}\ \forall n$. Since the latter is increasing in s_n, the lower bound is most restrictive for the firm with the largest market share s_{max}. Comparing this lower bound with the lowest price that induces zero profits $c - \alpha\bar{a} - \alpha(1-\alpha)\bar{a}$ shows that the latter is always larger.

Therefore, deviation (ii) is not binding in this case. Third, if $f_1 + \bar{a} \geq v$, I get another lower bound at $f_1 \geq c - \alpha\bar{a} + \frac{\eta\alpha}{s_n}(v-c) - \alpha(1-\alpha)\bar{a}$. This is most restrictive for the firm with the smallest market share s_{min}. Thus, the latter case imposes a lower bound on prices and thereby imposes minimal positive shrouding profits of $\eta\alpha(v-c)$ in the last two cases respectively.

Thus, deviation (i) induces an upper bound on prices and (ii) can induce a lower bound if $f_1 + \bar{a} \geq v$. In the latter case, shrouding equilibrium profits are always strictly positive and above $\eta\alpha(v-c)$.

Note that there can be no equilibrium in which firms play mixed strategies with a continuous distribution function. When firms mix on some interval with a continuous distribution function, conditional on prices of this interval occurring, the probability of having the same prices is zero and continuation profits are zero as well. Thus, standard Bertrand arguments such as those in the proof of Proposition 2 establish the usual contradiction.

There can, however, be shrouding equilibria in which firms mix over a finite number of prices, each price being played by each firm with positive probability. These prices must be within the range derived above, for otherwise (i) or (ii) above is a profitable deviation. Since continuation profits cannot be larger as when all firms coordinate on the same price with probability one, and the largest such price is given by $f_1 = c - \alpha\bar{a} + \frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a}$, profits must be below $s_n\frac{s_{min}}{1-s_{min}}\alpha(1-\alpha)\bar{a} + s_n\alpha(1-\alpha)\bar{a}\ \forall n$. At the same time, shrouding profits must be at least $\eta\alpha(v-c) > 0$ if total prices for naives are larger than v and zero otherwise for each firm.

B.5 Proof of Proposition 5

Step 1: Period 2 The following Lemma summarizes results on continuation equilibria. Afterwards, I study the first period.

Lemma 4 [Period 2 with Disclosure Policy]
An Equilibrium with shrouding in period 2 exists if and only if shrouding occurs in period 1. Shrouding occurs in period 2 either with probability one or with probability zero. When shrouding occurs, both customer types pay a total price of c and naives a hidden fee $\bar{a}$. Profits are zero in any continuation equilibrium.

Proof of Lemma 4.

First, I analyze continuation equilibria given shrouding occurs in period 1. By the exact same argument as in the proof of Proposition 3, continuation equilibrium profits are zero whenever some firm unshrouded in period 1.

Suppose prices were shrouded in period 1. Then continuation equilibrium profits must be zero conditional on shrouding and unshrouding. Suppose otherwise. Note that whether shrouding or unshrouding occurs, firms have symmetric information on customers and can charge those that were naive and sophisticated in period 1 separately in period 2. The markets for consumers who were naive or sophisticated in period 1 can therefore be treated as separate markets in period 2. For consumers that were sophisticated in period 1, the market is a standard Bertrand market and the results follow immediately. Recall that sophisticates are unaffected by shrouding. For the market for consumers that were naive in period 1, the argument is similar to the one used in the proof on Lemma 3(i) Step (II). Take the firm that earns the largest strictly positive profits conditional on either unshrouding or shrouding. If these profits occur conditional on shrouding, take the supremum for which these profits occur and denote it by $\bar{f}$. For positive profits to occur, each competitor must set larger prices with positive probability. I.e., competitors set prices in $[\bar{f}, \bar{f} + \epsilon)$ with positive probability for each $\epsilon > 0$, or $\bar{f}$ would be shifted upwards. But then competitors can increase their profits conditional on shrouding discretely by shifting probability mass from $[\bar{f}, \bar{f} + \epsilon)$ slightly below $\bar{f}$. Since losses conditional on unshrouding are below ϵ, this deviation is strictly profitable for some ϵ small enough, a contradiction. If the largest profits occur conditional on unshrouding the same argument applied to total prices applies. Thus, expected profits are zero for all customers conditional on shrouding and unshrouding. In particular when firms shroud with probability one, a firm's demand is independent of $\bar{a}$ and hence any firm sets $a_{n2} = \bar{a}$, and standard Bertrand arguments applied to each market imply that $f_{n2}^{soph} = c$ and $f_{n2}^{naive} = c - \bar{a}$. When shrouding occurs with probability zero, all consumers pay $f_{n2}^{soph} = f_{n2}^{naive} = c$ since all are aware of hidden fees, whether they can avoid them or not.

I study unshrouding incentives next. When firms shroud with probability one, all consumers pay a total price equal to marginal costs. Unshrouding and undercutting total prices for competitors' unavoiding naive customers reduces total prices below marginal costs and can therefore not profitably attract these customers. I now establish that shrouding either occurs with probability one or with probability zero. Suppose otherwise. Recall that firms earn zero profits in expectation whether shrouding or unshrouding occurs. When shrouding occurs, customers that were naive in period 1 must pay a transparent price below marginal cost and a hidden fee of $\bar{a}$. If this was not so, a firm could earn strictly positive profits by setting prices for customers that were naive in $t = 1$ of $c - \epsilon$ and $\bar{a}$ for some $\epsilon > 0$ small enough. This would marginally reduce profits on these customers when unshrouding occurs but discretely increase profits when shrouding occurs. Naives of period 1 therefore purchase at a transparent price below c when shrouding occurs and firms earn zero expected profits from them. But when unshrouding occurs, the share of naive customers in period 2 drops discretely to $\eta\alpha$ and with it the share of naives of period 1 that pay the hidden fee in period 2. Since these customers pay transparent fees below c and profits are zero when shrouding occurs, firms must earn strictly negative profits with these prices when unshrouding occurs. Thus, these firms are better of by unshrouding with probability one and setting transparent prices to c and hidden fees to zero, a contradiction. □

Step 2: Period 1

By Lemma 4, continuation profits are zero independent of first-period behavior. Hence, the setting is the same as in period 1 of Proposition 2.

From now on, results are proven for the setting described in the main text with all naives avoiding hidden fees after unshrouding ($\eta = 0$)

B.6 Proof of Proposition 7

Relative to Proposition 3 and 4, the incentives to unshroud have changed. First, I derive the shrouding condition for shrouding equilibria in period 2. Note that naives can avoid hidden fees after unshrouding so that they cannot be profitably attracted by unshrouding. Given shrouding occurred in period 1 and all firms have a positive customer base, the shrouding-equilibrium prices are the same as in proposition 3. When unshrouding occurs in t=2, a share $(1 - \lambda)$ of the old naives remain naive. The situation is the same when these consumers learn about hidden fees, i.e. when naiveté in period 1 is not a perfect predictor of naiveté in period 2. But past naiveté remains an informative signal that competitors do not have. Hence, conditional on unshrouding firms can guarantee themselves only profits of $s_n\alpha(1-\lambda)(1-\alpha)\bar{a}$ by setting naive-customer prices to $c - (1 - \lambda)\alpha\bar{a}$ and to c for sophisticates and new customers. Since this is strictly smaller than shrouding profits, the argument in the proof of Lemma 3, part (i) still applies accordingly and firms prefer shrouding over unshrouding when shrouding occurs with positive probability.

Second, I identify the most profitable deviations from a shrouding equilibrium path in period 1. There are three candidates: firms could unshroud without changing prices, firms could unshroud and undercut competitors or they could continue to shroud and undercut competitors.

At a given price f_1 that is charged by all firms, shrouding in $t=1$ is profitable if

$$s_n(f_1+\alpha\bar{a}-c)+s_n\alpha(1-\alpha)\bar{a}\geq s_n(f_1+(1-\lambda)\alpha\bar{a}-c)+s_n(1-\lambda)\alpha(1-(1-\lambda)\alpha)\bar{a},\quad \forall n. \tag{15}$$

For positive λ, this condition is equivalent to $\alpha \leq \frac{2}{2-\lambda}$, which holds for all λ. Note that after shrouding in period 1, the second period becomes equivalent to a model with a share of $\widetilde{\alpha}=(1-\lambda)\alpha$ of naive consumers and no option to unshroud. This induces equilibrium profits as in a shrouding equilibrium with a share of naives of $\widetilde{\alpha}$.

Unshrouding and undercutting a price f_1 is not profitable if

$$s_n(f_1+\alpha\bar{a}-c)+s_n\alpha(1-\alpha)\bar{a}\geq (f_1+(1-\lambda)\alpha\bar{a}-c)+(1-\lambda)\alpha(1-(1-\lambda)\alpha)\bar{a},\quad \forall n. \tag{16}$$

While simply undercutting is no deviation if

$$s_n(f_1+\alpha\bar{a}-c)+s_n\alpha(1-\alpha)\bar{a}\geq (f_1+\alpha\bar{a}-c)+(1-\lambda)\alpha(1-\alpha)\bar{a},\quad \forall n \tag{17}$$

it can be easily shown that the last condition is more restrictive for all $\lambda > 0$. It follows immediately that this condition is equivalent to

$$f_1\leq c-\alpha\bar{a}+\frac{s_n}{1-s_n}\alpha(1-\alpha)\bar{a}-\frac{1-\lambda}{1-s_n}\alpha(1-\alpha)\bar{a},\quad \forall n. \tag{18}$$

Since the r.h.s. is increasing in s_n, the largest price at which no firm has an incentive to undercut is given by the r.h.s evaluated at the smallest market share s_{min}.

The lower bound of the interval for equilibrium profits is given by the smallest price that earns firms nonnegative profits when all firms play this price with probability one.

B.7 Proof of Proposition 8

The only difference to the proofs of Propositions 3, 4 and Corollary 1 is the upper bound of the interval on which prices f_{n2}^{new} and f_{n2}^{naive} are mixed. By choosing f_{n2}^{new}, firms now attract the new arriving customers as well. Despite $f_{n2}^{soph}=c$, firms earn positive profits from $f_{n2}^{new}=c-\epsilon$, since marginal losses from sophisticates are offset by positive margins from newly arrived naives. Thus, competition drives f_{n2}^{new} down until n does not benefit from attracting new customers, i.e. until $((1-\gamma)+(1-s_n)\gamma(1-\alpha))(f_{n2}^{new}-c)+(1-\gamma)\alpha\bar{a}\leq 0, \forall n$, which results in $f_{n2}^{new}\leq c-\frac{(1-\gamma)}{(1-\gamma)+(1-s_n)\gamma(1-\alpha)}\alpha\bar{a}, \forall n$. For $N=2$, this pins down the interval as stated in the Proposition. For $N>2$, note that for each n, all $\hat{n}\neq n$ jointly have to choose $f_{\hat{n}2}^{new}$ to make n indifferent between an interval of naive-customer prices. Since this must hold for all n, they need to mix on the same interval and thus we get $[c-\alpha\bar{a}, c-\frac{(1-\gamma)}{(1-\gamma)+(1-s_{max})\gamma(1-\alpha)}\alpha\bar{a}]$.

The rest follows is as in Propositions 3 and 4.

B.8 Proof of Proposition 9

The proof is the same as for Proposition 3,4 and Corollary 1, except for one difference: the smaller share of naive customers in period 2. Here, all new-customer prices $f_{n2}^{new}<c-\sigma\alpha\bar{a}$ are never played in equilibrium with positive probability since they result in negative profits from new customers. This induces mixing of f_{n2}^{naive} and f_{n2}^{new} on $[c-\sigma\alpha\bar{a}, c]$. Second period profits become $\pi_{n2}=s_n\sigma\alpha(1-\alpha)\bar{a}$.

B.9 Proof of Proposition 10

In this proof, I construct an equilibrium for the T-period model in which shrouding occurs in each period.

Lemma 5 : Assume the customer bases of all firms is non-empty in $t-1$. Shrouding can occur in t if and only if shrouding occurred in $t-1$ and no customer-type switches in $t-1$. (Given shrouding in $t-1$, if customers switch in $t-1$, unshrouding occurs with probability 1 in t and profits become zero.)

Proof. Obviously, shrouding in t requires shrouding in all periods before t, otherwise consumers are educated about hidden fees in t. Let there be shrouding in $t-1$. Suppose that at least one firm attracted at least one customer type of her competitor in $t-1$. Then this firm is perfectly informed about her competitor's customers and the competitor thus earns zero profits in t. He is therefore indifferent between unshrouding or not. Due to the same equilibrium selection logic as in Proposition 4, i.e. only unshrouding being robust to the presence of unavoiding naive customers, unshrouding occurs with probability one. Similarly, if shrouding occurs, we get a contradiction if some customers switched in the past. □

Lemma 5 shows that switching of customers induces unshrouding in the subsequent period and therefore zero profits until period T. Hence, firms have a strong incentive not to let customers switch in order to maintain shrouding equilibria with positive profits.

Since the control and state variables are the same as in the two period model, Lemma 3 and Lemma 5 give the profits in period T of shrouding equilibria, namely $\pi_{nT} = s_n\alpha(1-\alpha)\bar{a}$. Denote by V_{nt} the continuation value of shrouding of firm n at the beginning of period t when the firms continue to shroud until T. We know from Lemma 5 that the continuation profit in all other cases is zero. Note that $V_{nT} = \pi_{nT} = s_n\alpha(1-\alpha)\bar{a}$.

Lemma 5 tells us as well that in each shrouding equilibrium path with shrouding in each period, no customer switches the firm until period T and each firm has a positive customer base. Denote one firm by n and the other $\hat{n}$. Then I can find the range of naive customers' prices f_{nt}^{naive} that prevents $\hat{n}$ from undercutting her competitor with $f_{\hat{n}t}^{new}$: Undercutting attracts all of n's customers once, earning $s_n(f_{nt}^{naive} + \alpha\bar{a} - c) + 0$, but induces a loss of future shrouding profits $0 + \delta V_{\hat{n}t+1}$.[38] Hence, all prices $f_{nt}^{naive} \leq \bar{f}_{nt}^{naive} \equiv c - \alpha\bar{a} + \frac{\delta}{s_n} \cdot V_{\hat{n}t+1}$ are not undercut by $\hat{n}$ and we therefore get $f_{nt}^{naive} = \min\{v, \bar{f}_{nt}^{naive}\}$.

Similarly, I can find the lowest new-customers price $f_{\hat{n}t}^{new}$that is undercut by n with f_{nt}^{soph} in order to prevent switching of any customer: $f_{\hat{n}t}^{new} < f_{nt}^{soph}$ makes n loose her sophisticated customers and gives $s_n\alpha(f_{nt}^{naive} + \bar{a} - c) + 0$ while undercutting and choosing $f_{nt}^{soph} = f_{\hat{n}t}^{new}$ in order to prevent switching gives $s_n\alpha(f_{nt}^{naive} + \bar{a} - c) + s_n(1-\alpha)(f_{\hat{n}t}^{new} - c) + \delta V_{nt}$. Hence, all $f_{\hat{n}t}^{new} \geq \bar{f}_{\hat{n}t}^{new} \equiv c - \frac{\delta}{s_n(1-\alpha)} \cdot V_{nt+1}$.
Using V_{nT}, it can be easily shown that $\bar{f}_{\hat{n}t}^{new} \leq \bar{f}_{nt}^{naive}$ for all $T > t > 1$. Therefore, we get prices for these periods of $f_{nt}^{naive} = f_{nt}^{soph} = \min\{v, \bar{f}_{nt}^{naive}\} = f_{\hat{n}t}^{new}$. Profits in period t become $\pi_{nt} = \min\{s_n(v + \alpha\bar{a} - c), \delta V_{\hat{n}t+1}\}$. This allows us to compute the shrouding condition in period t

$$V_{nt} \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v - c\}, \quad \forall n \text{ and } t > 1 \tag{19}$$

Using $V_{nt} = \pi_{nt} + \delta V_{nt+1}$ and the definition of π_{nt}, it is straightforward to show that V_{nt} is decreasing in t. Hence, $V_{nT} \geq \alpha\eta \min\{(1-\alpha)\bar{a}, v - c\}$ implies that all the other shrouding conditions are satisfied.

In period 1 the same argument applies as for Corollary 1 to pin down first period prices.

[38] I do not write the profits of $\hat{n}$ from her own customers in t since they are the same in each case.